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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

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VOLUME LXX OCTOBER 1929 NUMBER 3

THE CONTOURS OF HELIUM LINES IN STELLAR SPECTRA

By C. T. ELVEY

ABSTRACT

The contours of *He* 4472 and *He* 4388 have been obtained for twenty-eight stars from spectrograms made with the three-prism spectrograph of the Yerkes Observatory, and analyzed with the registering microphotometer. The contours can be represented by an equation of the simple exponential type.

The presence of the Stark effect is shown by the asymmetry of *He* 4388 and by the "forbidden" line λ 4470 of helium in the violet wing of *He* 4472.

"Dish-shaped" contours of the lines in the spectra of 16 λ Aql., 4 γ Cor., 19 q Tau., 44 ι Ori., and 22 λ Cep. indicate axial rotation of these stars.

By Milne's theory, logarithms of the number of atoms responsible for producing an absorption line are computed for various assumptions of the physical properties of the stars. These are shown in the form of curves by plots of log $N_0^{(s)}$ against the temperature.

The number of atoms NHf is determined for each star with use of Unsöld's theory. To eliminate the Stark effect and the broadening due to rotation, NHf is determined by comparing the areas of the observed contours with those given by theory. A value of 0.1 was arbitrarily chosen for f, and the resulting numbers of atoms have been compared with the curves computed from Milne's theory. The observations support the assumption that the coefficient of absorption varies with the pressure, rather than that it is a constant. The observations indicate pressures of 783 dynes/cm² to 9270 dynes/cm² for g equal to 10^3 cm/sec.,² the temperatures being from 10,000° to 30,000°. For g equal to 10 the pressures are 0.1 as large.

The contours of absorption lines in stellar spectra are functions of a variety of causes, each of which has its own particular effect upon the spectral line. These causes are some of the physical properties of the star and its atmosphere, and if it is possible to unravel the phenomena that produce the various contours of the absorption line in a spectrum, he will have in some detail the constitution of the outer part of the stars. The most probable causes of the broadening of spectral lines in a star are: (1) intramolecular electric fields, (2)

axial rotation of the star, and (3) the abundance of the atoms producing the absorption line.

The presence of the Stark effect in stellar spectra has been discussed by O. Struve.[1] He has shown that the widths of the hydrogen lines are probably due to intramolecular fields of 10^3-10^4 volts per centimeter. Also, that fields of this strength would show their presence in their effect upon other spectra, notably that of helium. The following observations are noted as indicating the Stark effect: The presence of wide and of narrow lines in the same spectral subdivisions and their correlation with absolute magnitude, the narrow lines appearing in the giant stars. The fluctuating Stark effect produces asymmetrical lines which cause shifts in the positions of the lines. The line at λ 4470 in the spectra of B-type stars is identified as the "forbidden" line of helium ($2p-4f$) which appears in electric fields.

A discussion of the axial rotation of stars has been presented by Shajn and Struve.[2] They have computed the contours of the spectral lines in rapidly rotating stars, assuming that the original contours of the lines are the same as those in the spectrum of the moon. The effect of rotation depends upon the initial width and depth of the line. For the same depth the narrowest lines are affected the most; that is, the line is widened at the expense of the intensity of the center of the line. In the case of lines of the same width the deepest lines are the most affected. Thus, a narrow deep line in a rapidly rotating star, such as a very short-period binary, would become a very wide and shallow line. A velocity of rotation of 50 km/sec. should be measurable without much difficulty.

Unsöld[3] has derived theoretical contours for absorption lines. In the case of a stellar line the contour may be represented by the equation

$$(\lambda-\lambda_0)^2=NHf\,\frac{2\pi e^4\lambda_0^2}{3m^2c^4}\left(\frac{1-I}{I}\right),$$

where λ is the wave-length of a point on the contour of the line; λ_0, the wave-length of the line; f, the oscillatory strength; N, the num-

[1] *Astrophysical Journal*, **69**, 173, 1929.

[2] *Monthly Notices*, **89**, 222, 1929.

[3] *Zeitschrift für Physik*, **46**, 765, 1928.

ber of atoms per cubic centimeter; H, the height of the atmosphere in centimeters; m, e, the mass and charge of the electron; I, the intensity of the point on the contour expressed in percentage of absorption of the continuous spectrum; c, the velocity of light.

By fitting the equation to the observed contour of the line one can determine the value of NHf, the number of atoms effective in producing the line. Owing to the distortions of the contour by the rotation of the star and by mol-electric fields, one is unable to make a direct comparison of the theoretical and the observed contours of the line. However, one can integrate the equation for the area of the theoretical contours by assuming various values of NHf and by comparing these areas with the areas of the observed contours.

The contours of the helium lines λ 4472, diffuse triplet $2p^3-4d^3$, and λ 4388, diffuse singlet $2P-5D$, were obtained from spectrograms taken with the three-prism Bruce spectrograph. The plates used were Eastman 40, and each was standardized with a "tube" sensitometer and developed in Eastman contrast developer, D-11, for five minutes. The scale of the spectrograms is 9.8 A per millimeter at λ 4472 and 9.1 A per millimeter at λ 4388. Each spectrogram was analyzed with a registering microphotometer of the thermoelectric type. The image of the spectrogram was projected upon the analyzing slit in front of the thermopile so that a strip of spectrum 0.049 A wide at λ 4472 and 0.046 A wide at λ 4388 was on the sensitive element at any one time.

In Figures 1, 2, 3, and 4 are plotted the contours of the helium lines λ 4388 and λ 4472 of twenty-eight stars, the abscissae being $(\lambda-\lambda_0)$ and the ordinates being the intensities expressed as percentages of absorption of the continuous spectrum. There are forty-eight spectrograms which are distributed among the spectral types as follows: Od, 1; Oe5, 4; Bo, 3; B1, 3; B2, 2; B3, 6; B5, 5; B8, 3; and B9, 1. The contours of 27 Canis Majoris were omitted in that the star will receive special attention in a future investigation. Six of the contours have been published by the author in a "Note on Otto Struve's Paper on the Stark Effect in Stellar Spectra,"[1] but are being repeated here for completeness.

Considering the mean contours of the red sides of the lines for

[1] *Astrophysical Journal*, **69**, 237, 1929.

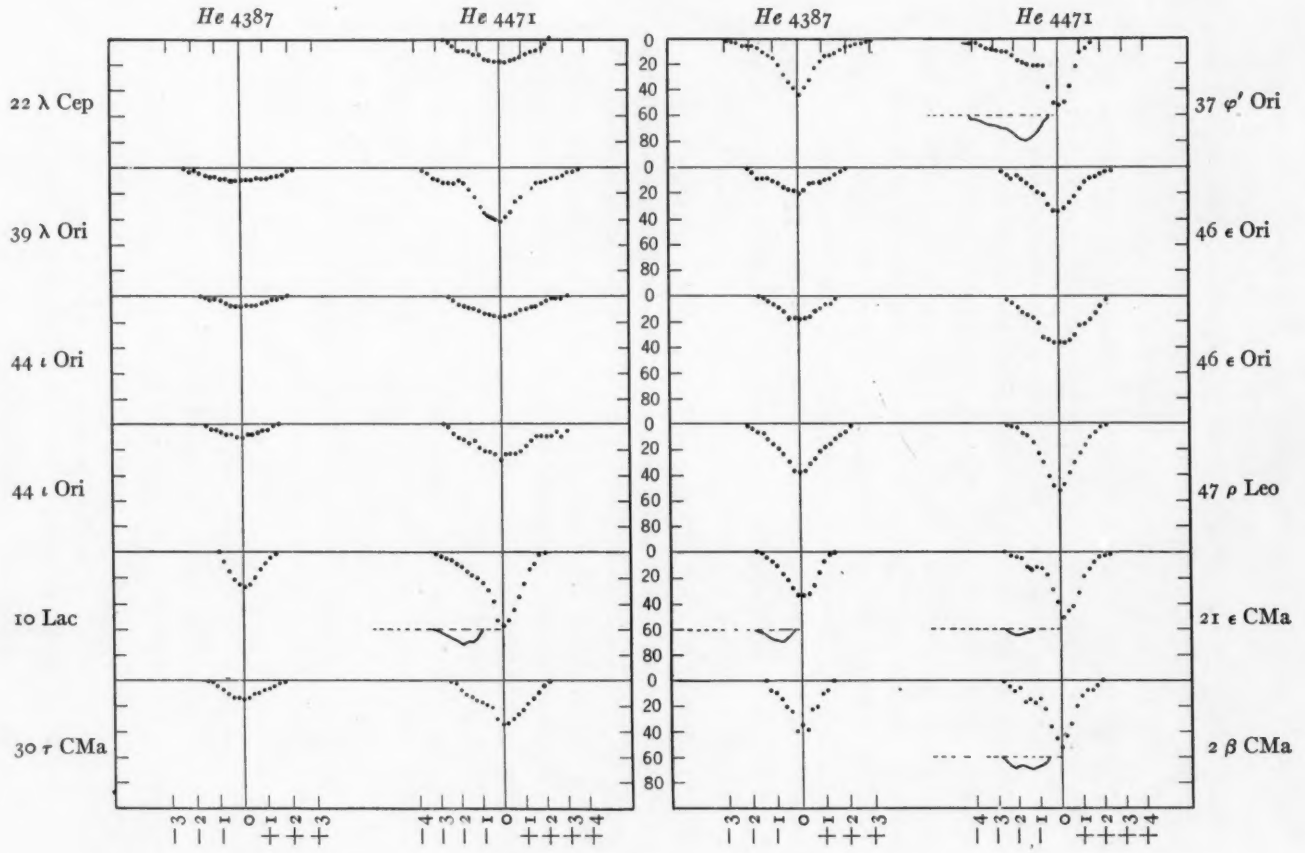

FIG. 1.—Contours of helium lines

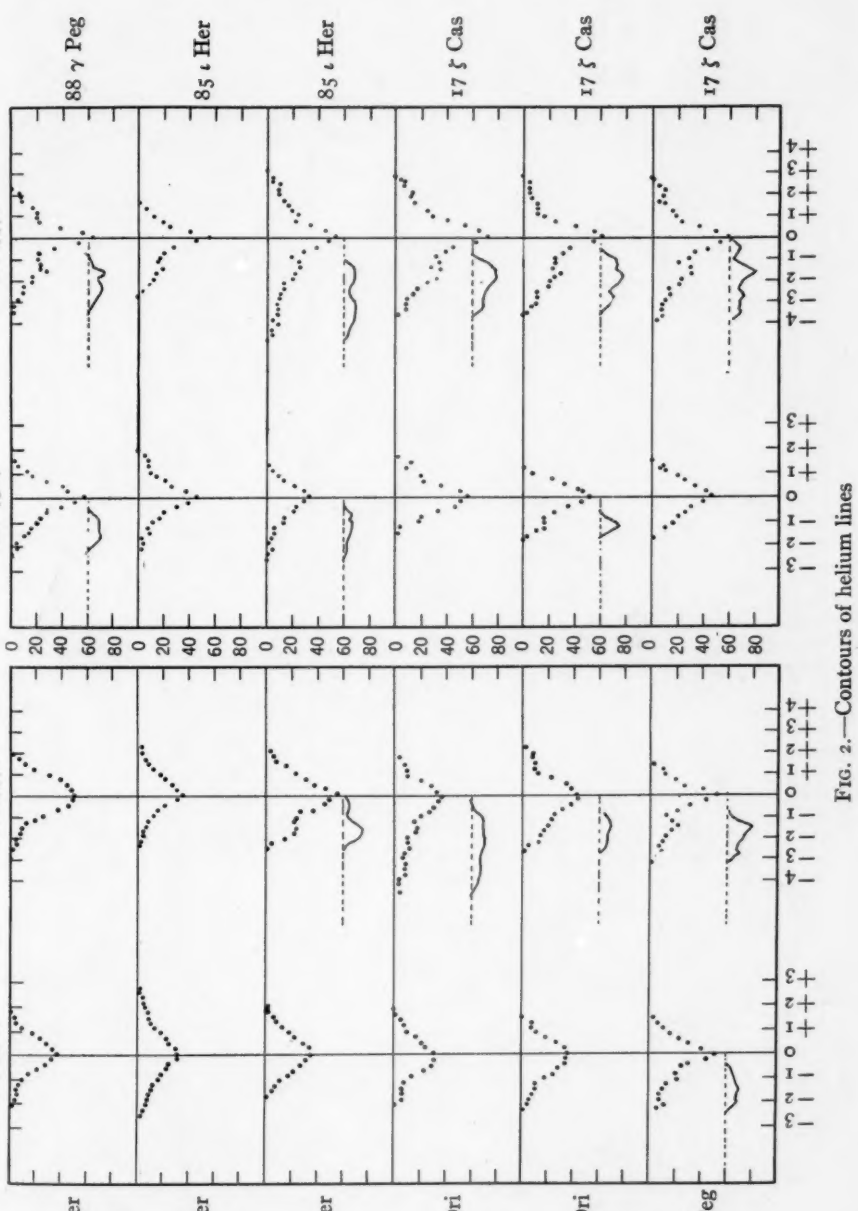

FIG. 2.—Contours of helium lines

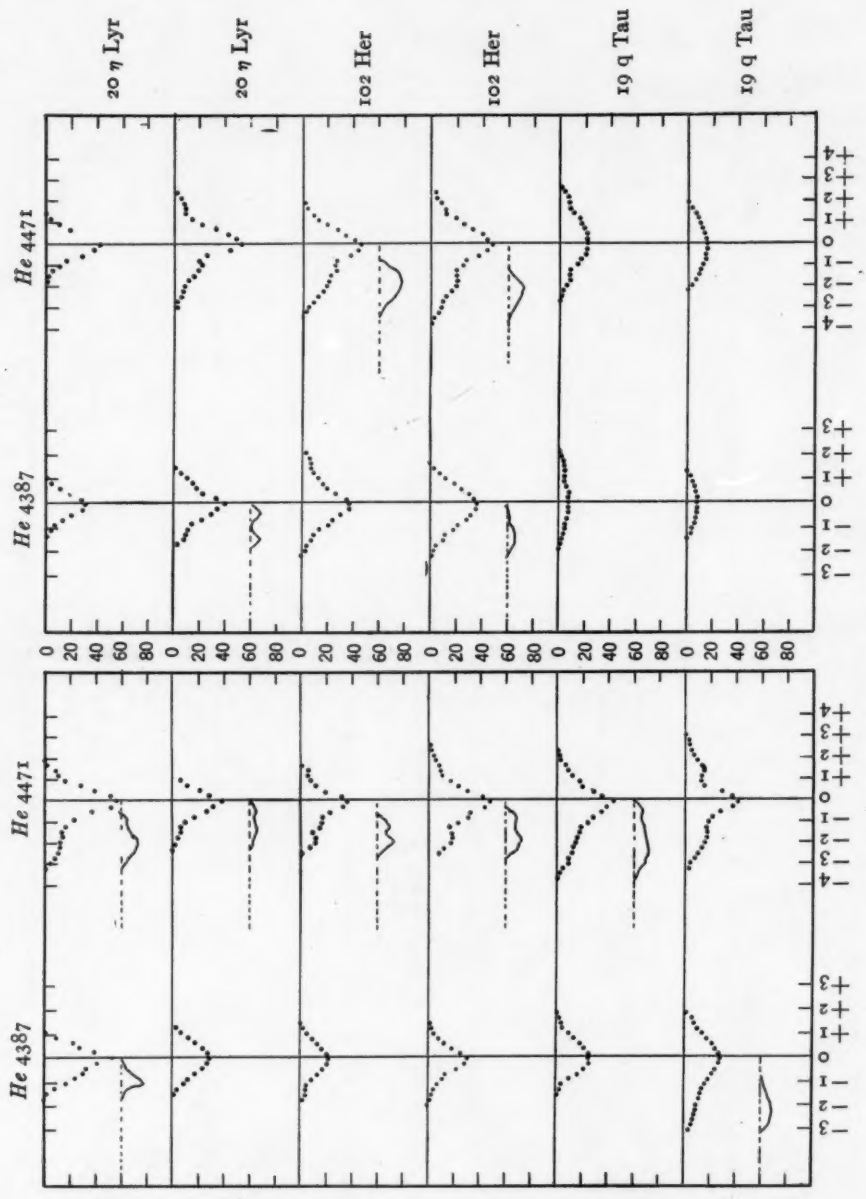

FIG. 3.—Contours of helium lines

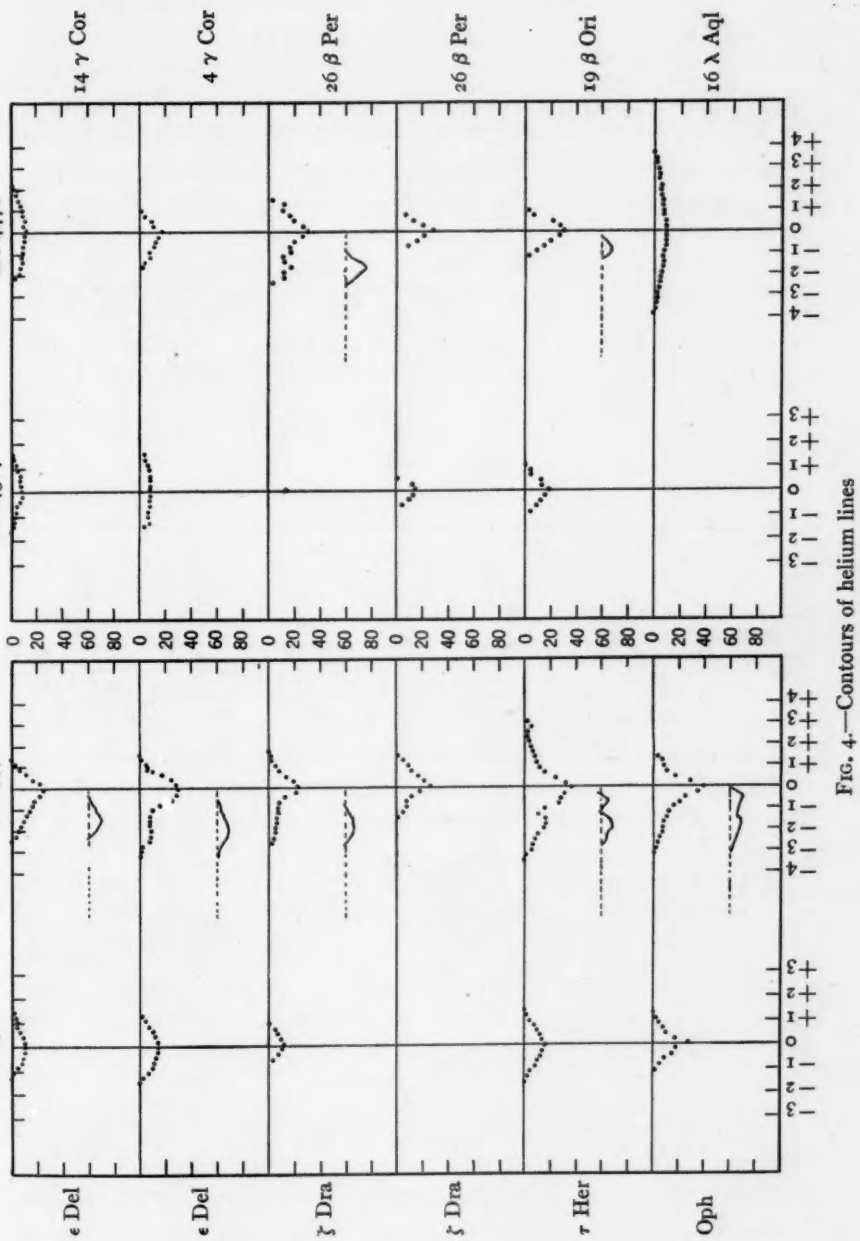

FIG. 4.—Contours of helium lines

each spectral subdivision one finds that the simple exponential equation fits the contours. This agrees with the writer's findings in

$$\frac{I}{I_0} = e^{-k(\lambda-\lambda_0)}$$

respect to the hydrogen lines in A-type stars. The fit with the equation is illustrated by Figure 5 in which the ratios of the intensity I of

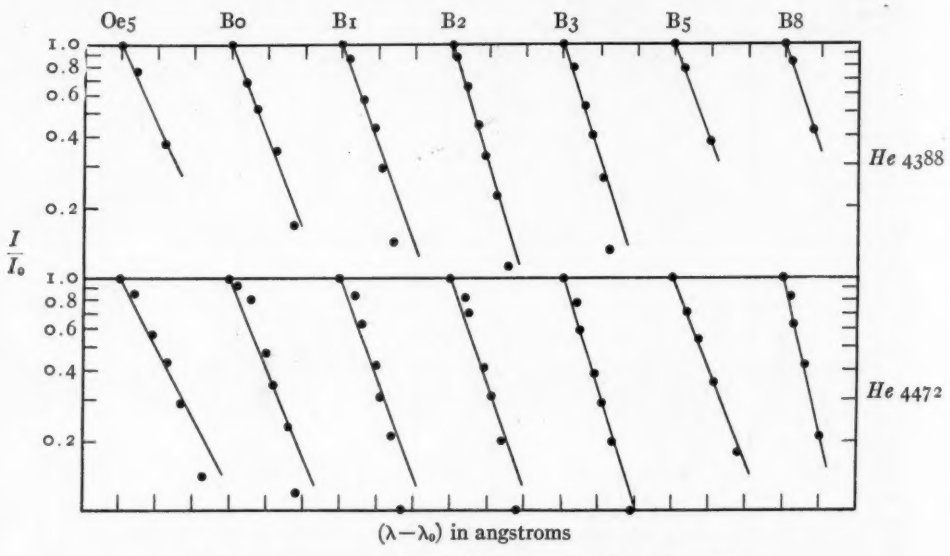

FIG. 5.—Contours of helium lines plotted on semi-log paper

any point to the intensity I_0 at the center of the line are plotted on the logarithmic scale of the semi-log paper and the distances of the points from the centers of the lines, the half-widths, in angstroms are plotted on the linear scale. In such a graph the simple exponential equation gives a straight line and the value of the exponent k in the equation may be readily obtained. These values are listed in Table I.

The central intensities of the helium lines λ 4472 and λ 4388 are listed in Table II for each star. The means for each spectral class have been taken with assignment of weights for each star equal to the number of spectrograms analyzed. These mean intensities have been plotted against the spectral types as shown in Figure 6*a*, the

dots representing *He* 4472 and the circles *He* 4388. *He* 4388 gives the sharper maximum and seems to be located quite definitely at type B2. The maximum for *He* 4472 is somewhat more uncertain, being about halfway between types B2 and B3. The line λ 4388, with an excitation potential of 21.12 volts, would be expected to have its maximum at a little earlier spectral type than the line λ 4472 with an excitation potential of 20.81 volts. From estimates of a much larger number of stars Struve[1] places the maxima of these two lines between types B1 and B2.

Struve has called attention, also, to the variation of the relative intensities of these two helium lines. The ratio of the central inten-

TABLE I

Type	$\frac{k}{4388}$	$\frac{k}{4472}$
Oe5	0.51	0.45
Bo	.43	.40
B1	.37	.36
B2	.33	.30
B3	.33	.32
B5	.40	.40
B8	0.23	0.35

sity of λ 4472 to that of λ 4388 was taken for each of the stars. These data given also in Table II. It is noticed that the ratio within any spectral class is nearly constant except for three individual stars. This is best shown diagrammatically by Figure 6*b*. The ratios are plotted as ordinates and the spectral types as abscissae. For each spectral type a vertical line is drawn to represent the range of the values of the ratio, excluding the exceptional ones which are plotted as dots. The weighted means for each spectral class are joined by lines. The star 39 λ Orionis (Oe5) has a ratio of 4.7 as compared with the mean of 2.2 for its class. The star 27 Canis Majoris (B5p) has a ratio of 3.6, and the mean of its class is also 2.2. This star is suspected by Struve[2] to be a very massive object. The other peculiar B5-type star, 67 Ophiuchi, has a somewhat low value of the ratio, it being 1.4.

[1] *Nature*, **122**, 994, 1928.

[2] *Astrophysical Journal*, **68**, 109, 1928.

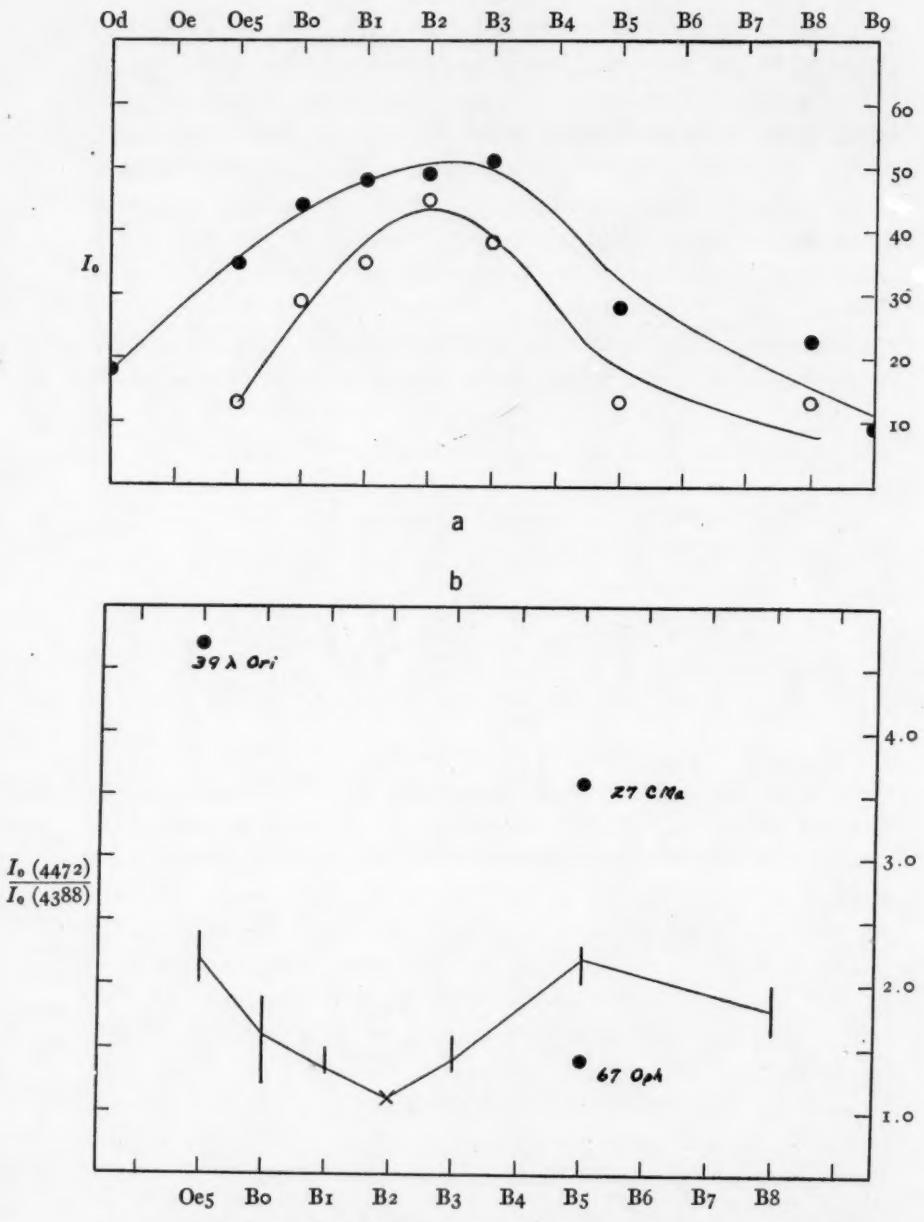

FIG. 6.—(*a*) Central intensities of helium lines; dots are *He* 4472 and circles are *He* 4388. (*b*) Ratios of central intensities of *He* 4472 to *He* 4388. The observations fall within the vertical lines except for the three stars indicated.

The "forbidden" line of helium, λ 4470, which makes its appearance in electric fields, is seen to be present on the violet wings of many of the contours of *He* 4472. In some cases there is only an asymmetry of the line while in others it is clearly resolved. The contour due to λ 4470 is shown by taking the excesses of the intensities

TABLE II

Star	Spec. Type	Wt.	I_{4472}	I_{4388}	$\frac{I_{4472}}{I_{4388}}$
22 λ Cep	Od	1	18	.....	.....
39 λ Ori	Oe5	1	42	9	4.7
44 ι Ori	Oe5	2	20	10	2.0
10 Lac	Oe5	1	57	24	2.4
30 τ C Ma	Oe5	1	34	14	2.4
37 φ′ Ori	Bo	1	50	42	1.2
46 ε Ori	Bo	2	36	19	1.9
47 ρ Leo	Bop	1	52	37	1.4
21 ε C Ma	B1	1	51	35	1.5
2 β C Ma	B1	1	52	40	1.3
44 ζ Per	B1	3	46	34	1.3
24 γ Ori	B2	2	40	35	1.1
88 γ Peg	B2	2	58	54	1.1
85 ι Her	B3	2	54	39	1.4
17 ζ Cas	B3	4	63	51	1.2
3 π^4 Ori	B3	2	47	35	1.3
45 ε Cas	B3	4	42	26	1.6
20 η Lyr	B3	2	47	35	1.3
102 Her	B3	2	47	37	1.3
19 q Tau	B5	2	18	8	2.3
2 ε Del	B5	2	28	12	2.3
22 ζ Dra	B5	2	25	12	2.0
22 τ Her	B5	1	39	17	2.3
67 Oph	B5p	1	39	28	1.4
27 C Ma	B5p	1	32	9	3.6
4 γ Cor	B8	2	14	8	1.8
26 β Per	B8	2	28	14	2.0
19 β Ori	B8p	1	31	19	1.6
16 λ Aql	B9	1	9	.....	.....

of the violet side of a line over those of the red side, and are shown as continuous lines in Figures 1–4. These lines in a few stars have been discussed in the author's note on Struve's paper (*loc. cit.*). The mean wave-length of the component λ 4470 as measured from the density-curves in sixteen stars is 4469.77 I.A. as compared with 4469.88 I.A. obtained by Struve[1] from the measurement of spectrograms, and with 4469.86 I.A. given by the writer in the note referred to above.

[1] *Ibid.*, **62**, 198, 1925.

The mean intensity of the "forbidden" line is 12 per cent absorption of the continuous spectrum.

The line λ 4388 is seen to be asymmetrically broadened toward the violet on many of the contours. This line should widen in this manner in mol-electric fields. If one considers the asymmetry as due to a second component but not resolved by the spectrograph, one can obtain an idea of its wave-length and intensity by taking the differences of the violet and red sides of the line. The wave-length is 4386.65 I.A. and its intensity is 10 per cent absorption of the continuous spectrum.

In Figures 1–4 there are a number of contours which are decidedly "dish-shaped." Comparing these dish-shaped contours with the theoretical contours of rapidly rotating stars obtained by Shajn and Struve (*loc. cit.*), we see that there is a marked similarity, so much so that we feel safe in stating that the stars 16 λ Aquilae, 4 γ Corvi, 19 q Tauri, 44 ι Orionis, and 22 λ Cephei are rotating rapidly. The dish-shaped lines seem to prefer the earlier and later spectral classes of the series rather than the middle types B1–B3. According to the theory of broadening from abundance of a spectral line, the earlier and later classes would have the narrowest lines and hence would be the most affected by rotation. Another possible explanation is that since rapid rotation weakens the central intensities of the lines the rotating stars may be put in spectral classes farther from the maximum than they would otherwise have been.

In a series of papers on ionization in stellar atmospheres E. A. Milne[1] uses the following definition of the intensity of an absorption line. A point Q on the contour of the line which has an intensity I, or an intensity ratio r, ratio of the residual intensity at the point to the intensity of the continuous spectrum, corresponds to a definite optical depth in the star. The half-width $(\lambda-\lambda_0)$ at this point is a measure of the intensity of the line since the number of atoms absorbing the radiation is proportional to $(\lambda-\lambda_0)^2$ (see Unsöld's formula above). In discussing problems involving the maxima of absorption lines one must determine the intensity-curves in the foregoing manner. Since different positions in the contours correspond to different optical depths in the star (the wings of a line are produced in the

[1] *Monthly Notices*, **89**, 3, 17, 1928.

low levels and the core of the line in the high levels), slightly different intensity-curves will be found for the various points on the contour.

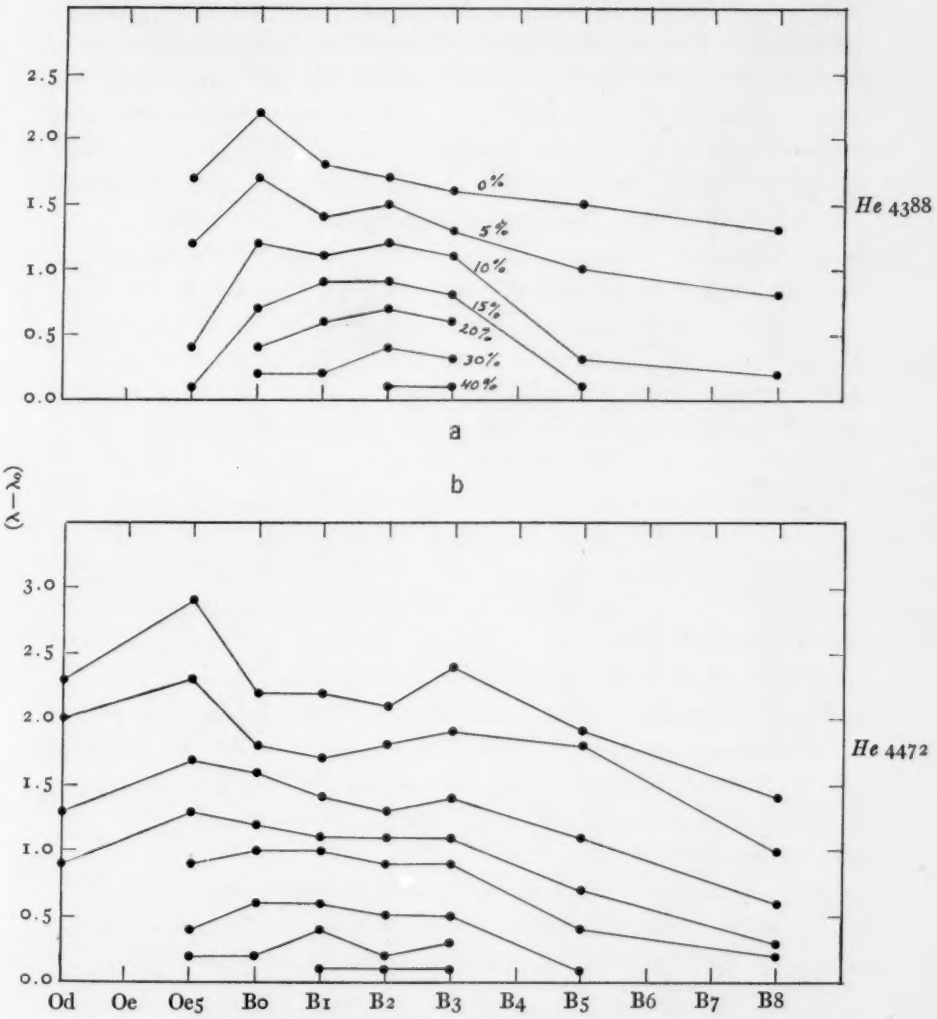

FIG. 7.—Half widths of *He* 4388 and *He* 4472 for points on the contours equal to 1, 5, 10, 15, 20, 30, 40, and 50 per cent absorption of the continuous spectrum.

The half-widths of the red sides of the helium lines were measured in each contour for the intensity ratios, $r = 0.60$, 0.70, 0.80, 0.85, 0.90, 0.95, and 0.99, which considered as percentages of absorption of the continuous spectrum are, respectively, 40, 30, 20, 15,

10, 5, and 1. The red sides of the lines were taken to eliminate the effect of the forbidden lines λ 4470 and λ 4386. The means for each spectral class were taken and the results are shown by Figures 7*a* and 7*b*. The half-widths in angstroms have been plotted as ordinates and the spectral classes as abscissae. The maxima for the various positions in the wings of the line λ 4472 are somewhat uncertain in that there is a tendency to form two maxima, at B3 and Oe5. The general trend of the maxima is a shift toward the earlier spectral types for the parts of the lines originating at the deeper levels in the atmospheres of the stars. This is in agreement with expectation.[1] In order to discuss the maxima of *He* 4472 for the lower levels it will be necessary to obtain more spectrograms of stars of spectral types earlier than Oe5. In the case of the line λ 4388 the curves are quite definite, the maxima progressing from B2.2 for the intensity ratio 0.70 to Bo.o for the extreme wing of the line, intensity ratio 0.99.

In the papers referred to above Milne has discussed the theory of thermal ionization as applied to a stellar atmosphere, taking into account the varying degree of ionization from level to level. He has derived formulae for the total number $N_r^{(s)}$ of atoms in a stellar atmosphere in a given state of ionization r and excitation s down to any depth in terms of the electronic pressure P at that depth. The principal parameter is the electronic pressure at the base of the layer and not the mean pressure throughout the layer as has been used in previous investigations. A mean value is taken for the temperature T, which probably varies only within narrow limits. Also, a constant value is taken for gravity, g, that is, general and selective radiation pressure is neglected.

His expressions for the "generalized Saha formulae" may be summarized in the following equations. The numbers of the equations are identical with the same equations in his papers.

$$N = \frac{\epsilon}{mg}\int_0^p dp = \frac{\epsilon p}{mg}, \tag{17}$$

$$N_0 = \frac{\epsilon}{mg}\int_0^p x_0\, dp, \tag{17'}$$

$$N_1 = \frac{\epsilon}{mg}\int_0^p x_1\, dp, \tag{17''}$$

[1] *Ibid.*, p. 34, 1928.

where N, N_0, and N_1 are, respectively, the total number of atoms of a given element, the number of neutral atoms, and the number of once-ionized atoms of that element down to the level where the total pressure is p; ϵ, the abundance of the element by weight; m, the mass of the atom; g, the value of gravity; x_0, x_1, the fractions of neutral and once-ionized atoms. If P is the electronic pressure at the level considered, then the ionization formula is

$$\frac{x_1}{x_0} P = K_1 , \tag{18}$$

where

$$K_1 = \frac{q_1}{q_0} \frac{(2\pi m_e)^{\frac{3}{2}}}{h^3} (kT)^{\frac{5}{2}} e^{-\frac{\chi_1}{kT}} . \tag{19}$$

Here m_e is the mass of the electron; χ_1 is the first ionization potential; q_0, q_1, the statistical weights of the neutral and once-ionized atoms in their normal states; and the other symbols have their usual meaning.

The method of attack on the problem is to express p, x_0, x_1 in terms of P and to evaluate the integrals (17′) and (17″), taking T constant. The values of p, x_0, and x_1 are different according to the conditions of the atmosphere. In the case of helium the conditions are satisfied in Milne's "*Problem III.—The atoms in question are just becoming ionized in the presence of an excess of atoms already once-ionized.*" The solutions of the integrals give

$$N = \frac{\epsilon}{mg} 2P , \tag{35}$$

$$N_0 = \frac{2\epsilon}{mg} \left[P - K_1 ln \left(1 + \frac{P}{K_1} \right) \right] , \tag{35′}$$

$$N_1 = \frac{2\epsilon}{mg} K_1 ln \left(1 + \frac{P}{K_1} \right) , \tag{35″}$$

where *ln* is the natural logarithm.

The ratio of the number $N_0^{(s)}$ of neutral excited atoms of statistical weight $q_0^{(s)}$ in the state $\chi_1^{(s)}$ to the number of neutral atoms N_0 is

$$A_0^{(s)} = \frac{N^{(s)}}{N_0} = \frac{q_0^{(s)}}{q_0} e^{\frac{-(\chi_1 - \chi_1^{(s)})}{kT}} . \tag{56}$$

Then the number of atoms in the state to absorb a given line is

$$N_0^{(s)}=\frac{2\epsilon}{mg}A_0^{(s)}\left[P_0-K_1 ln\left(1+\frac{P_0}{K_1}\right)\right]. \tag{78}$$

Instead of following his procedure of dealing with problems of maximum intensity of an absorption line Professor Milne suggested that I follow his modification of the method which is yet unpublished. The method is to take equation (78) and to compute $N_0^{(s)}$ for a series of values of T assuming various values of the unknowns. The unknowns are ϵ, g, and κ. The abundance can be taken as 5 per cent. If this quantity is in error it will affect all values of $N_0^{(s)}$ by a constant factor. Several values for gravity can be taken and a series of curves will be found which will show any effect of the absolute magnitudes of the stars. Two conditions are considered in regard to the coefficient of absorption; (1) κ is constant and (2) κ varies with the pressure, thus

$$\kappa=\frac{a'P}{(T/10^4)^{\frac{5}{2}}}.$$

Milne finds that 0.85 gives satisfactory results for the value of the constant a' in the case of the zinc lines in the sun.

To obtain the pressure one has the relationship of the optical depth to the coefficient of absorption and the pressure

$$\tau=\frac{1}{g}\int^{p}\kappa\, dp,$$

which gives in the case of Problem III

$$\tau_0=\frac{a'}{g(T/10^4)^{\frac{5}{2}}}P^2.$$

The optical depth τ_0 of the bottom of the layer is obtained from the contour of the line with use of the relation

$$\tau_0=\frac{\frac{4}{3}r}{r+1},$$

r being defined as above. Milne considers this expression of the optical depth to be more satisfactory than that given by equation (42).[1]

[1] *Monthly Notices*, **89**, p. 12.

With the aid of the two equations for the optical depth the pressure can be calculated for various values of a', g, and T.

Figure 8 shows the results of the calculations, giving theoretical curves for the numbers of helium atoms producing the line λ 4472. The temperatures are plotted as the abscissae and the logarithms of the number of atoms in a given state as the ordinates. No account has been taken of second ionization. Curves 1, 2, and 3 show the variation of $N_0^{(s)}$ with a' which equals 4.25, 0.85, and 0.17, respectively. The optical depth is 0.613 and the surface gravity g is 10^4 cm/sec.^{2} These values of the constant a' give entirely too few atoms to account for the intensities of the lines. If the value of a' is chosen as 10^{-3}, which corresponds very closely with the theoretical value of the constant a in the formula[1]

$$\kappa = \frac{aP}{(kT)^{\frac{5}{2}}},$$

a more reasonable value is obtained for the number of atoms.

Curves 5 and 5′ are the representation of the equation for a' equaling 10^{-3}, $\tau_0 = 0.613$, and $g = 10^3$ and 10 cm/sec.^{2}, respectively. These curves show that on the low-temperature side of the maximum of intensity there is a variation of the number of atoms with surface gravity. In other words, there is an effect of absolute magnitude for the stars of spectral classes earlier than the maximum. For g equaling 10^3 P varies from 783 to 9270 dynes for temperatures 10,000° to 30,000°, and for g equal to 10 the range is 78–927 dynes.

In order to make a comparison with the case where the coefficient of absorption is assumed as a constant and equal to the mean value, curves have been computed for g equal to 10^3 and 10^4 cm/sec.^{2} These are shown by curves 4 and 4′, respectively. In this case the number of atoms in a giant star is less than the number in a dwarf at the same temperature. Observations are contrary to this. Also, there is no difference between the giants and dwarfs on the low-temperature side of the maximum. This is again contrary to observation since a supergiant A-type star such as α Cygni has helium lines in its spectrum while they are absent in the spectra of the normal stars of the type.

[1] *Ibid.*, p. 37, 1928.

In order to make a more reliable comparison between the theory and observation, the numbers of atoms absorbing *He* 4472 and *He* 4388 have been determined for the stars in this investigation, by the above-mentioned method of comparing the areas of the observed contours with those given by Unsöld's formula. The values of *NHf* are given in Table III. For the helium line λ 4472 a probable

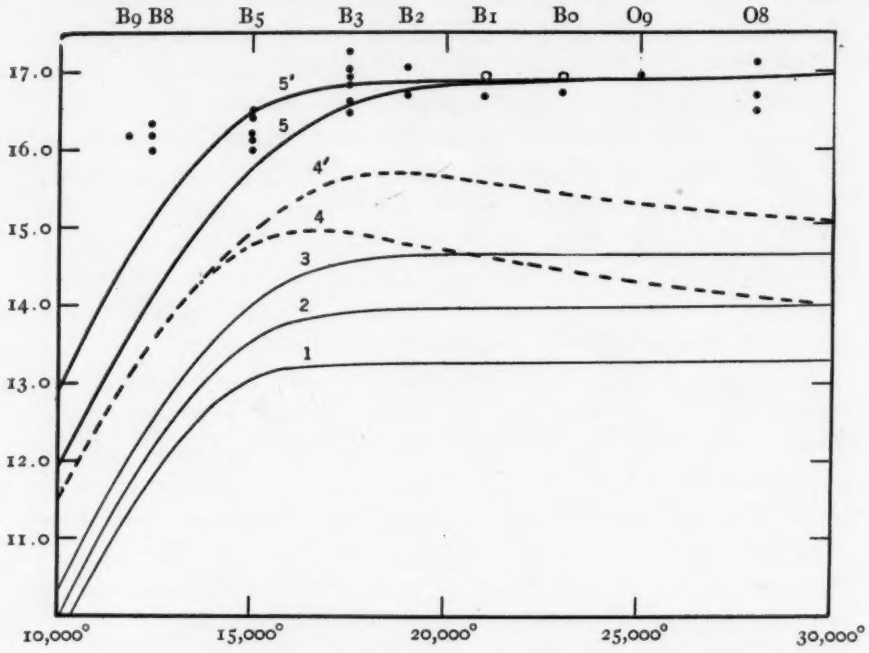

FIG. 8.—Curves of logarithms of numbers of atoms of *He* 4472 representing various assumptions. The dots are the observations.

value of 0.1 was arbitrarily chosen for *f*, and the logarithms of the resulting numbers are plotted in Figure 8. The open circles represent points where two stars had the same number of atoms. The choice of a temperature scale is the same as that given by Russell, Dugan, and Stewart[1] and extended to the O-type stars.

The values of *NH* for types B8 and B9 are the only ones that deviate markedly from the theoretical curves. If the observations are to be trusted the stars would seem to have extensive atmos-

[1] *Astronomy*, II, p. 753.

pheres, or are wrongly classified; or an increase of 2000° in the temperature scale is indicated.

TABLE III

Star	Spectral Type	log *NHf*	
		He 4472	*He* 4388
22 λ Cep	Od (O6e)	15.22	.....
39 λ Ori	Oe5 (O8)	16.13	15.01
44 ι Ori	Oe5 (O8)	15.50	14.74
10 Lac	Oe5 (O9)	15.94	15.16
30 τ CMa	Oe5 (O8)	15.70	14.92
37 φ′ Ori	Bo	15.74	15.78
46 ε Ori	Bo	15.94	15.18
47 ρ Leo	Bop	15.94	15.86
21 ε CMa	B1	15.94	15.40
2 β CMa	B1	15.70	15.40
44 ζ Per	B1	15.94	15.50
24 γ Ori	B2	15.70	15.34
88 γ Peg	B2	16.06	15.70
85 ι Her	B3	16.02	15.54
17 ζ Cas	B3	16.27	15.78
3 π^4 Ori	B3	15.46	15.30
45 ε Cas	B3	15.64	15.22
20 η Lyr	B3	15.78	15.30
102 Her	B3	15.94	15.60
19 q Tau	B5	15.40	14.80
2 ε Del	B5	15.16	14.74
22 ζ Dra	B5	15.00	14.55
22 τ Her	B5	15.50	14.84
67 Oph	B5p	15.22	15.02
4 γ Cor	B8	15.00	14.84
2 β Per	B8	15.34	14.61
19 β Ori	B8p	15.16	14.84
16 λ Aql	B9	15.16	.....

I am very much indebted to Professor E. A. Milne for his advice on the methods of comparing the observations with the theory, and to Professor Otto Struve for his many suggestions.

Yerkes Observatory
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ELEMENTS UNIDENTIFIED OR DOUBTFUL IN THE SUN

SUGGESTED OBSERVATIONS[1]

By CHARLES E. ST. JOHN

ABSTRACT

Elements unidentified in the sun.—Argon, actinium, arsenic, gold, bismuth, bromine, chlorine, fluorine, mercury, iodine, iridium, krypton, masurium, neon, osmium, phosphorus, protoactinium, polonium, radium, rhenium, radon, selenium, tantalum, tellurium, thorium, uranium, xenon, and the rare earths, holmium, illinium, lutecium, terbium, and thulium.

Elements suggested for laboratory investigation.—Arsenic, iridium, osmium, phosphorus, tantalum, thorium, thallium, uranium and the rare earths. Study of these elements promises immediately useful information.

Data required.—(*a*) For immediate application to problems of identification we need accurate determinations of the wave-lengths of critical lines and a study of their relative intensities and appearance under varied conditions of excitation. (*b*) For general purposes we require precision wave-lengths throughout the spectral ranges of the different elements, preferably in vacuum, and analyses of the respective spectra.

The identification and characteristic behavior of the Fraunhofer lines in the sun's spectrum deserve special attention and study, not only because the sun is of more immediate importance to us than any other star, but also because its position midway between the terrestrial laboratories and the stars is unique and therefore strategic. The intensity of radiation reaching the earth enables us to use instruments of observation and analysis equal to those in the laboratory and thereby furnishes the means of obtaining accurate data and testing quantitatively theories and assumptions. It was in anticipation of the advantages assured by a co-ordinated attack through laboratory, sun, and stars that the threefold organization at Mount Wilson arose.

In the preparation of tables of data on the solar spectrum astrophysicists of today have at hand methods and lines of attack undreamed of a generation ago. These may vary for individual elements, but for all there should be reasonable agreement in the position of solar and laboratory lines, so-called coincidence. We know now that exact coincidence is the exception and, when exact, points to possible errors of observation. As a rule, the departures from

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 385.

coincidence are positive for λ sun *minus* λ arc in vacuum and of the order of 0.01 A. Though close agreement in wave-length offers usually a first line of attack, this is not always the case. Not infrequently the approach has come through an analysis of the spectrum which has shown just what to look for and where to look; for example, the oxygen doublet in the infra-red.

Identifications of elements represented by few solar lines resting upon coincidences alone are subject to much uncertainty through accidental agreements in wave-length. In such cases considerations based upon atomic theory have been the determining factors.

Spectral lines are associated with the transitions of an atom between two energy-levels or spectroscopic terms which depend upon the configurations of the outer electrons. In the case of absorption the work required to raise the energy of the atom in its normal state to the lower of the two levels of the transition is the excitation potential for the line. It is measured in "volts," the "volt" being the energy acquired by an electron in falling through a difference of potential of 1 volt. For any element the absorption lines arising from transitions between the lowest and the next higher level, the resonance lines, are the strongest and the most probable to occur in the sun, provided they are in the region of the spectrum accessible to observation, viz. to the red of λ 2975 A. If these are not within the accessible region, then those from the next level are the most probable. The excitation potential is the important factor in determining the probability of the presence of a line in the spectrum.

The identification in the sun of the stronger lines of a multiplet facilitates the identification of the weaker members through the knowledge of their positions and relative intensities. It is a great gain to know for what to look and with what we need not bother. If the strong lines of a multiplet are absent or only weakly represented, the faint members may safely be neglected.

Fifty-eight of the 90 known elements appear with varying degrees of probability to be constituents of the sun's atmosphere (Table I). A brief examination of the elements not yet identified in the sun will indicate the possibilities for increasing the identified list and the new observational data most likely to prove of value for the purpose.

In the group of inert gases (Table II) the systematic decrease of the ionization and excitation potentials and the shift of the resonance region to longer wave-length with the increase of atomic number illustrate the progression so marked in the second half of columns IV–VIII in the periodic table and appearing later in Tables III, IV, and V.

For the inert gases the resonance lines begin far in the ultra-violet, and even for radon do not fall within the accessible region;

TABLE I

ELEMENTS IDENTIFIED IN THE SUN AND THEIR POSITION IN THE PERIODIC TABLE

(Unidentified Elements in Parentheses)

I	II	III	IV	V	VI	VII	VIII			
H	.....	.....	.....	.....	.....	.....	*He*			
Li	*Be*	*B*	*C*	*N*	*O*	*(F)*	*(Ne)*			
Na	*Mg*	*Al*	*Si*	*(P)*	*S*	*(Cl)*	*(A)*			
K	*Ca*	*Sc*	*Ti*	*V*	*Cr*	*Mn*	....	*Fe*	*Co*	*Ni*
Cu	*Zn*	*Ga*	*Ge*	*(As)*	*(Se)*	*(Br)*	*(Kr)*			
Rb	*Sr*	*Y*	*Zr*	*Cb?*	*Mo*	*(Ma)*	....	*Ru*	*Rh*	*Pd*
Ag	*Cd*	*In*	*Sn?*	*Sb*	*(Te)*	*(I)*	*(Xe)*			
Cs	*Ba*	*La**	*Hf*	*(Ta)*	*W*	*(Re)*	....	*(Os)*	*(Ir)*	*Pt*
(Au)	*(Hg)*	*Tl?*	*Pb*	*(Bi)*	*(Po)*	85	*(Rn)*			
87	*(Ra)*	*(Ac)*	*(Th)*	*(Pa)*	*(U)*	.....	.....			

* RARE EARTHS

Ce	*Pr*	*Nd*	*(Il)*	*Sa*	*Eu*	*Gd*
(Tb)	*Dy*	*(Ho)*	*Er?*	*(Tu)*	*Yb?*	*(Lu)*

and comparisons with solar absorption lines in the accessible regions (fifth column, Table II), do not show agreement in wave-length for any of these elements. The question of inexact data does not enter, for the wave-lengths of neon, argon, krypton, and xenon are known to the highest precision.[1] The non-appearance of these elements in the sun's spectrum is probably due to the combined effect of high excitation potential and very moderate abundance. The great majority of solar lines are due to the metals, and for them the mean

[1] Burns, Meggers, Merrill, *Bureau of Standards, Scientific Papers*, **14**, 765 (No. 329), 1919; Meggers, *ibid.*, **17**, 193 (No. 414), 1921.

excitation potential for neutral atoms is 1.319 volts, with a maximum, in the case of lead, of 5.771 volts. The maximum line intensity for lead is -2, the next to the faintest observable. The inert gases, with still higher excitation potentials, would need to be far more abundant than lead to produce even the faintest observable lines. Hence absence of their lines from the solar spectrum is not surprising and does not forbid their presence in the solar atmosphere with an abundance comparable to that in the earth.

TABLE II

INERT GASES

Atomic Number	Element	I.P.	Resonance Lines	Accessible Lines	E.P.	Identification
2.....	Helium	24.48	584.4	5875	20.871	*He*
10.....	Neon	21.47	735.95 743.73	3369–8495 55 lines	16.54	.....
18.....	Argon	15.69	1048.21 1066.62	3948–8521 50 lines	11.50	.....
36.....	Krypton	13.9	1164.90 1235.87	4273–7601 18 lines	9.9	.....
54.....	Xenon	12.1	1295.65 1469.67	4500–4923 12 lines	8.2	.....
86.....	Radon...	.....	2842.1* 2892.7*	4166.5† 4203.3†	.....	.....

* Strongest ultra-violet lines (S. Wolf, *Zeitschrift für Physik*, **48**, 790, 1928).

† Strongest and most persistent lines in the accessible region (T. Royds, *Philosophical Magazine*, **17**, 202, 1909; H. E. Watson, *Proceedings of the Royal Society*, A, **83**, 50, 1910).

For radon (radium emanation) accurate wave-lengths are not available. Radon is the most radioactive element and has an average life of 5.57 days.[1] It is an immediate product of radium, and its radioactive change evolves helium, polonium, and eventually lead. Unless in equilibrium with its mother-substances, it is rapidly dissipated; hence until thorium, uranium, and radium are detected in the sun, it would be vain to look for radon.

The remaining inert gas is helium. In spite of its high excitation potential of 20.871 volts, the helium line λ 5875 occurs in absorption in highly disturbed regions, and with other helium lines it is promi-

[1] F. Soddy, *Interpretation of Radium* (London, 1922), p. 113.

nent in the flash spectrum. More surprising still is the presence of the enhanced line λ 4686 in the flash spectrum, requiring as it does an excitation of 72.57 volts. In view of its high excitation potential, it is obvious that helium must be present in abundance, as is indeed shown by the great height, 8000 km, to which it extends on photographs of the flash spectrum. The behavior of helium suggests causes of excitation in the sun's atmosphere not yet brought to light.

Passing to the left in the periodic table to the halogens, we find a similar situation (Table III). The resonance lines are in the inaccessible ultra-violet region, and only for fluorine and chlorine have

TABLE III

HALOGENS

Atomic Number	Element	I.P.	Region of Resonance Lines	Most Probable Lines	E.P.
9.....	Fluorine	17.3	951–958	6239–6413 6774–6909 7256–7745	12.5
17.....	Chlorine	12.8	1335–1689	7980–8585 8948–9702	8.5
35.....	Bromine	11.4	1232–1633	.....	7.5
53.....	Iodine	10.2	1234–2062	.....	6.0

arc lines of the next energy-level been identified. These, the most probable lines, are in the red and infra-red. Their excitation potentials are high in comparison with those of other non-metals represented in the sun by weak lines alone. They show no systematic agreement in position with solar lines, though occurring in spectral regions where agreements should be easily evident since the solar lines are not closely crowded. The arc wave-lengths have recently been measured to three decimals for fluorine[1] and to two places for chlorine.[2] Lines of bromine and iodine of the indicated excitation potential are theoretically possible in the visible region, but have not been observed.

The data on band spectra of the halogens are too meager for

[1] H. Dingle, *Proceedings of the Royal Society*, A, **113**, 323, 1926; **117**, 407, 1928; T. L. de Bruin, *Amsterdam Academy Proceedings*, **30**, 944, 1927; I. S. Bowen, *Physical Review*, **29**, 231, 1927.

[2] L. A. Turner, *Physical Review*, **27**, 401, 1926.

profitable discussion. There is no evidence, and no present prospect of evidence, based on their spectra to show the presence of the halogens in the sun; nor are they found in the stars. In view of their abundance in the earth, their absence from the sun would be surprising. The failure to detect them is not inconsistent with their presence in the sun, for they would need to be very abundant in order to produce observable absorption lines in accessible spectral regions, as their excitation potentials are so high. The ozone in the upper reaches of the earth's atmosphere blocks appeal to the lines from the lowest energy-levels.

TABLE IV

OXYGEN GROUP

Atomic Number	Element	I.P	Resonance Lines	Accessible Lines	E.P.	Identification
8.....	Oxygen	13.56	1358	7774 8446	9.106 9.479	*O* Triplet *O* Doublet
16.....	Sulphur	10.31	1900	9224	6.496	*S* Triplet
34.....	Selenium	9.4	2050	.....	6	.....
52.....	Tellurium	8.7	2375	.....	5.6	.....
84.....	Polonium	.....	Radioactive, average life 202 days			.....

Of the oxygen group (Table IV), both oxygen and sulphur have been included as present in the sun's atmosphere. For selenium and tellurium the most probable lines would be farther to the red than for oxygen and sulphur, but nothing is known of their arc spectra in this region. McLennan, McLay, and McLeod[1] give one line of neutral tellurium in the ultra-violet at λ 3175.15 of intensity 3 on a scale of 20 R for the strongest resonance line λ 2142.75. There is an unidentified solar line at λ 3175.166 of intensity -3, giving, sun *minus Te* arc, +0.016 A, but without additional support this near coincidence for a single line can hardly be accepted as identifying it in the sun.

Polonium, being radioactive, would not be expected as long as thorium and uranium, the mother-elements, are unidentified, particularly in view of its short average life of 202 days.

[1] *Philosophical Magazine*, 4, 486, 1927.

The evidence for atomic nitrogen (Table V) in the sun's atmosphere is given by the three strongest lines of the P—D quartet in the infra-red, λλ 8680.35, 8683.61, and 8686.38. These lines are from the next to the lowest energy-level, and within the errors of observation they agree in position with solar lines of intensities −2 to −1 not otherwise identified. Some doubt of the solar identification existed owing to the apparent absence of λ 8683 from the solar spectrum, but this has been dissipated, as the line has recently been measured upon a half-dozen spectrograms taken by Mr. Babcock in his extended investigation of the infra-red solar spectrum.

TABLE V

NITROGEN GROUP

Atomic Number	Elements	I.P.	Resonance Lines	Most Probable Lines	E.P.	Identification
7.....	Nitrogen*	14.2	1200	8680 8683 8686	10.0	*N*
15.....	Phosphorus	10.5	1781	9000–11,000	7	.....
33.....	Arsenic	9.6	1972	3000	6.2	.....
51.....	Antimony	8.35	2311.50	3100	2.024	*Sb*
83.....	Bismuth	7.25	3067.69	2989–4722	0.000	.....

* C. C. Kiess, *Journal of the Optical Society of America*, **11**, 1, 1925; Merton and Pilley, *Proceedings of the Royal Society*, A, **107**, 411, 1925; Hopfield, *Physical Review*, **27**, 801, 1926; O. S. Duffenback and R. A. Wolfe, *Physical Review*, **34**, 409, 1929; S. B. Ingram, *ibid.*, p. 421, 1929.

For an element with the high excitation potential of nitrogen, viz., 10 volts, only a small fraction of its atoms in the sun's atmosphere would be excited to the energy states required for the production of its absorption lines. The presence in the solar spectrum of lines due to nitrogen implies therefore that the total amount is large. The abundance of nitrogen indicated in the sun is consistent with its abundance in the earth and its wide distribution in stars and nebulae.

The resonance lines of phosphorus[1] are in the far ultra-violet, and no arc lines are known in the visible region. There are, as for all of this group, possible lines in the infra-red. Dhavale identifies as

[1] Mohler and Foote, *Physical Review*, **15**, 321, 1920; Ruark, Mohler, Foote, and Chenault, *Bureau of Standards, Scientific Papers*, **19**, 463 (No. 490), 1924; M. O. Saltmarsh, *Philosophical Magazine*, **47**, 874, 1924; McLennan and McLay, *Transactions of the Royal Society of Canada* (3d ser.), **21**, sec. 3, 63, 1927; Dhavale, *Nature*, **123**, 799, 1929.

phosphorus two lines in the infra-red with arc lines deduced from theory, but they have not been observed in the laboratory.

The remaining elements of the nitrogen group require a more detailed examination and will be considered in connection with Tables VI, VII, and VIII.

Only four arc lines of arsenic (Table VI) are known in the accessible region. The wave-lengths, except for λ 2990, are recent measures by Miss Ware on plates by Mr. King. The difference sun *minus* arc for λ 2990 is far too large to be accounted for by accidental errors

TABLE VI

ARSENIC*

Resonance Lines	Accessible Lines				Δλ Sun−Arc	Identifications
	Arc		Sun			
	λ I.A.	Int.	λ I.A.	Int.		
1972.....	2990.99	2	.861	−1	−0.129 A	.....
			.101	0	+ .111	*Ni*
2349.....	3032.850	4	.868	−3	+ .018	.....
2780.....	3075.320	2	.356	0	+ .036	*OH−Sc*+
	3119.591	4	.505	3	− .086	*Fe*
			.679	1	+0.088	*OH−Cr*

* Ruark, Mohler, Foote, and Chenault, *Bureau of Standards, Scientific Papers*, **19**, 463 (No. 490), 1924.

of measurement. The residual sun *minus* arc for λ 3032 is consistent in sign and magnitude with its identification as arsenic. If the solar line is due to arsenic, it fixes a maximum intensity of −3 for arsenic lines in the sun, and λ 3119 should also be present. It would, however, fall midway between two solar lines, but so far from either that it should be possible to detect it, as in this region lines of the same solar intensity and still nearer to even stronger solar lines are given by Rowland. The available evidence for arsenic as a probable constituent of the sun's atmosphere does not appear strong enough to justify its inclusion, as it rests upon one out of four lines. A resurvey of this region of the solar spectrum under high dispersion would be necessary for a definite conclusion. It is known from other evidence that not all the faint lines have been recognized in this region, and

the possibility that the line was missed holds the question still open.

The arc wave-lengths of antimony (Table VII) are measures from 1912. Though to three places and on the international system, they are of doubtful value, as the difficulties in such determinations were not at that time recognized as fully as later. Professor Kayser in his *Hauptlinien*[1] rounds them off to one and two places of decimals. Two of the three strong reversed lines show good agreement with solar lines of reasonable intensity, but for one the identification is questionable. Of the four weaker lines one is absent and three are masked

TABLE VII

ANTIMONY*

ARC			SUN		Δλ SUN−ARC	IDENTIFICATION IN SUN
λ I.A.	Int.	E.P.	λ I.A.	Int.		
3029.824	4R	2.024	.832	−2	+0.008 A	*Sb*
3232.537	4R	2.279	.548	−3N	+ .011	*Sb Raie ultime*
3267.497	4R	2.0204	.540	−3N	+ .043	*Sb? Raie ultime*
3383.150	3	.....	.....	.....	.....	.....
3637.830	3	.....	.874	4	+ .044	Masked by *Fe*
3722.787	3	2.024	.760	0	− .027	Masked by *Ce*
4033.562	3	.....	.586	1	+0.024	Masked by *Mn*

* Ruark, Mohler, Foote and Chenault, *loc. cit.*, p. 476; McLennan and McLay, *loc. cit.*

or blended with other elements. The main evidence for the presence of antimony in the sun rests upon the near coincidence of the strong arc lines of low excitation—the *raies ultimes*—with solar lines of appropriate intensity and not otherwise identified. Wave-lengths of greater accuracy are desirable, particularly for the arc in vacuum.

In bismuth (Table VIII) the fine structure[2] is manifest to a high degree. This renders comparison with solar lines difficult. The resonance line λ 3067 should furnish the most decisive criterion for the presence of bismuth because of its low excitation, great intensity, accurate wave-length, and simplicity of structure—two equal components or reversal. The absence of this *raie ultime* seems to eliminate bismuth. The strong lines have six or more components

[1] H. Kayser, *Tabelle der Hauptlinien der Linien-spektra Aller Elemente*, 1926.

[2] Nagaoka and Mishima, *Proceedings of the Tokyo Imperial Academy*, **2**, 249, 1926; Goudsmit and Back, *Zeitschrift für Physik*, **43**, 321, 1927.

spread over a range of 0.25 A, and the individual components are too weak to appear in the sun. In the case of narrow separations one would expect the components to form a broadish line, but of this there seems to be no evidence in the sun in the case of bismuth.

For elements giving rich arc spectra with few prominent lines the most probable lines to show in the sun are the strong absorption

TABLE VIII

BISMUTH*

ARC			E.P.	SUN		Δλ SUN − ARC	IN SUN	REMARKS
λ I.A.†	Int.	Comp.		λ I.A.	Int.			
2989.045	9R	2	1.410	.936	0N	−0.109	*Sc*	Under-water;‡ strong
				.196	−1	+ .151	*Cr*⁺	
2993.344	9R	4	1.410	.355	−2	+ .011	.....	Under-water; weak
3024.646	8R	6–12	1.906	.652	−2Nd?	+ .006	.....	Under-water; strong
3067.732	9R	2 or R	0.000	.789	1	+ .057	.....	Under-water; very strong
3067.712§	.....	.....	.....	.....	.....	+ .079	.....	*Raie ultime*
3397.208	5R	6	1.906	.222	1	+ .014	*Fe*	Under-water; rev.
3397.216‖	.....	.....	.....	.....	.....	+ .006	.....	.....
3510.853	6R	6	1.906	.847	5	− .006	*Ti*⁺	Under-water; rev.
3510.899	.....	.....	.....	.....	.....	− .052	.....	.....
4722.190	10	.....	.....	.165	3	− .025	*Zn*	.....
4722.542	10	6	1.410	.468	−3	− .074	.....	.....
4722.831	10	.....	.....	.....	.....	.....	.....	Under-water; sharp
4722.501‖	.....	.....	.....	.468	−3	−0.033	.....	*Raie ultime*

* Ruark, Mohler, Foote, and Chenault, *Bureau of Standards, Scientific Papers*, **19**, 482 (No. 490), 1924.

† Offerman, Dissertation, Bonn, 1920.

‡ L. and E. Bloch, *Journal de physique*, **3**, 309, 1922.

§ Measured as two components in the low density arc with wave-lengths ending in decimals .672 and .756; and as a reversed line in furnace with decimal .710 (King).

‖ Weighted mean of components.

lines in the spectra of the under-water spark, as they arise from low energy-levels. As stated by Meggers and Laporte,[1] the characteristics of the under-water spark are: (*a*) All lines observed as "reversals" in arc emission spectra are always observed reversed in the under-water spark. (*b*) In so far as the temperature classification of emission lines has been studied in the electric furnace, it is evident that the lines ordinarily absorbed in under-water spark spectra are always the lines of lowest temperature class.

The strongest lines in the spectra of osmium, iridium, and platinum are in the ultra-violet. Those in Table IX are the strongest

[1] *Physical Review*, **28**, 645, 1926.

lines in under-water absorption in the accessible region. There are no recent wave-length data for osmium and iridium. Those here used are from the *Handbuch der Spectroscopie* adjusted to the I.A. system. For platinum the recent measures are by McLennan and McLay.[1]

The spectroscopic data show little if any evidence for osmium and iridium in the sun. The strong osmium line agrees in position with a solar line of intensity −3, but the solar line is identified with a ruthenium line (E.P. 0.000), and λ 3058, the line of lowest energy-level, is

TABLE IX

OSMIUM-PLATINUM GROUP

Atomic Number	Element	λ I.A.	Intensity		E.P.	In Sun		Δλ Sun − Arc	Identification
			E	Abs.		λ I.A.	Int.		
76.....	Osmium	3018.059	4	10	.....	.983	−2N	−0.076	*Ni*
		3040.959	5	10	.....	.918	2	− .041	.....
		3058.670	6	20	0.00	.707	0	+ .037	.....
		3156.262	6	10	.....	.273	2	+ .011	*Fe*
		3262.308	6	10	.51	.354	1	+ .046	*Fe*
		3267.959	6	15	.....	.911	2	− .048	*Fe*+
		3301.572	7	25	.....	.580	−3	+ .008	*Ru* E.P. 0.000
77.....	Iridium	3068.90	4	6	.35	.947	−1	+ .05	.....
		3220.81	8	5	.35	.776	−1	− .03	Persistent
						.836	−1	+ .03	.....
78.....	Platinum	2997.965	7R	30	.096	.965	0	.000	*Pt*
		3042.634	4R	10	.102	.663	3	+ .029	*Pt? Fe*
		3064.710	6R	40	.000	.696	1	− .014	*Pt*
		3301.859	10	5	0.811	.870	−3	+0.011	*Pt*

apparently absent. As ruthenium is represented by more than a score of lines, maximum intensity 0, the solar line at λ 3301.580 is probably due to ruthenium alone. For iridium the evidence is weaker still. For both elements the lack of accurate wave-length data leaves a factor of uncertainty. In the case of platinum the recent wave-length measures permit more definite conclusions. Of the four lines from the lowest energy-levels and strongest in under-water absorption, three agree in position with solar lines whose intensities match well the emission and absorption intensities. The fourth is possibly blended with the strong iron line.

For thallium (Table X) there are no measures of wave-length later than 1904. The existing measures are to two decimal places and

[1] *Transactions of the Royal Society of Canada* (3d ser.), **20**, sec. 3, 201, 1926.

quite discordant. For the solar comparisons the means of arc and spark measures are used. Of the lines strong in the arc and in the under-water spark λ 3519 was identified by Rowland as *Tl?* and λ 3775 as *Tl.* The most persistent lines are λ 3775 and λ 5350, and for these the correspondence with solar lines of consistent intensity is within the errors of the measures. The other lines add little or nothing to the evidence. Thallium is, however, retained as probably present. Where a line was identified by Rowland and later evidence renders the identification doubtful, it has usually been retained, but questioned. Only in the lack of positive evidence or because of a more probable source for the line has an identification

TABLE X

THALLIUM, ATOMIC NO. 81

ARC			UNDER-WATER* SPARK ABSORPTION	IN SUN		Δλ SUN−ARC	IDENTIFICATION
λ I.A.	Int.	E.P.		λ I.A.	Int.		
3229.77	10R	0.962	R sharp	.796	0	+0.026	*Fe?*
3519.24	10R	.962	R strong	.100	−3Nd?	− .140	.....
				.392	−3	+ .159	.....
3529.41	8R	.962	R strong	.355	−1	− .055	.....
3775.74	10R	.000	R strong	.712	−2	− .031	*Tl?*
5350.50	10R	0.962	R strong	.505	−3N	+0.005	*Tl?*

* Bloch, *Journal de physique*, 3, 309, 1922.

been rejected. The fine structure of λ 5350, a complex line,[1] has been much studied. The observed separations and relative intensities of the strong components are shown in Table XI.

The positive identification of such complex lines in the sun would furnish the means of studying conditions in the sun's atmosphere from an unusual angle. For more certain identification of thallium in the sun, determinations of wave-lengths in vacuum are needed, together with further work on the conditions of excitation.

There remain in the main body of the periodic table the new elements, masurium and rhenium, for which our information is too meager; a few heavy elements, tantalum, gold, mercury, thorium, and uranium, for which precise wave-length data are lacking; and lastly, the strongly radioactive elements, radium, actinium, and protoactinium.

[1] Rowland, *Philosophical Magazine*, 36, 67 n., 1893.

The arc spectra of tantalum, thorium, and uranium are very rich in lines, of which few are outstanding. Their spectra have not been analyzed, but it is probable from comparison with homologous elements that their excitation is low and that their persistent lines are in the observable region.[1] For them observations of spectra in the under-water spark offer the most promising means of identification.

In the case of gold, E.P. 4.60, and mercury, E. P. 4.65, the resonance lines are in the ultra-violet, λ 2427 and λ 2536, respectively. The present measurements of their wave-lengths in the accessible region are wanting in accuracy. Such as they are, they seem to furnish no evidence of the probable presence of these heavy elements in the sun, but as the presence of mercury has been suggested[2] as well as that of tantalum, thorium, and uranium, it is desirable that wave-length data of high precision be obtained for at least the important lines, in order that comparison with solar lines may yield unequivocal results.

TABLE XI

SEPARATION AND RELATIVE INTENSITY OF COMPONENTS OF THALLIUM λ 5350

SEPARATION	RELATIVE INTENSITIES*		AUTHORITY
	Blue	Red	
0.12 A	1	0.50	Michelson
.21	1	.50	Perot and Fabry
.10	1	.75	Barnes
.114	1	.25	Janicki
0.11	1	0.35	Nutting

* Michelson, *Philosophical Magazine*, **34**, 280, 1892; Perot and Fabry, *Annales de chimie et de physique*, **16**, 115, 1899; Barnes, *Astrophysical Journal*, **19**, 190, 1904; Nutting, *ibid.*, **23**, 64, 1906; Janicki, *Annalen der Physik*, **19**, 36, 1906.

CONCLUSIONS

The preceding survey indicates the direction that efforts to widen our knowledge of the constituents of the sun and to narrow the list of the unrecognized elements may profitably take. Fortunately, the requirements for these objects run concurrently with those desired for the analysis of spectra, and results of laboratory observation are of equal importance to the physicist and to the astronomer. In recent

[1] Russell, *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929.

[2] Stratton, *Astronomical Physics* (London, 1925), p. 41.

years the progress in spectral analysis has been amazingly successful, but there still remain important elements whose spectra have so far failed to yield to it, and in great measure it is these same elements for which data are lacking for a definite examination of the composition of the sun's atmosphere. These spectra, because of their inherent difficulty, have naturally been left to the last, but, in view of past successes, it is not too much to hope that they will soon come within the pale of the conquered elements. There is great need for wave-lengths of the highest precision to replace the earlier approximate measures, but of equal, if not greater, importance is the investigation of the effects due to varying the conditions under which the spectra are produced in the laboratory. This has been a most fruitful line of attack to gain a foothold from which to storm the spectral citadel. Many such methods of attack are recognized and used, but there remains always the possibility of the discovery of a new method, and it behooves the investigator to be ever on the watch for what seem to be anomalous or unexpected effects, and to endeavor to relate them to conditions that favor their production.

The outstanding gaps and deficiencies in related solar and laboratory data have been given in detail. They are summarized below, together with suggested lines of attack for those elements whose investigation promises to yield the most immediate results capable of filling the gaps and removing the deficiencies.

SUMMARY

Osmium and iridium.—Known strong lines are in the accessible region. For these elements accurately determined wave-lengths of a limited number of lines would suffice for the immediate purpose of comparison with solar lines, but the measurement of the complete spectra is of increasing importance. Further observation of the solar spectrum in the ultra-violet is needed.

Tantalum, thorium, and uranium.—The spectra probably have unrecognized persistent lines in the accessible region. For their identification the under-water spark offers a promising means. Precise wave-length measures are lacking for all.

Phosphorus.—Investigations of the infra-red in the laboratory and in the sun are indicated as the most promising means of finding

dependable evidence for phosphorus in the sun. Phosphor-bronze may prove advantageous for obtaining the arc spectrum.

Arsenic, antimony, bismuth, and thallium.—Re-examination of the solar spectrum in the extreme violet with high dispersion. Re-measurement of antimony, thallium, and bismuth spectra in the same region, preferably in vacuum, and a study of the relation of fine structure to excitation, in bismuth and thallium.

Rare earths.—These offer a very promising field of investigation. They are of astrophysical importance in the investigation of low levels in solar and stellar atmospheres. For five of them data are quite inadequate for comparison with the sun. They have been considered in a separate contribution.[1]

I take this opportunity to acknowledge my debt to Professor H. N. Russell for access to his manuscript in which he discusses the quantitative representation in the sun of the elements as identified in the *Rowland Revision*[2] recently compiled at Mount Wilson Observatory, and to Dr. I. S. Bowen, of the California Institute of Technology, for his helpful suggestions.

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[1] St. John and Moore, *Mt. Wilson Contr.*, No. 364; *Astrophysical Journal*, **68**, 93, 1928.

[2] Revision of Rowland's Preliminary Table of Solar Spectrum Wave-lengths, *Carnegie Institution of Washington, Publications*, No. 396, 1928.

RONCHI'S METHOD OF OPTICAL TESTING[1]

By J. A. ANDERSON and RUSSELL W. PORTER

ABSTRACT

1. *Ronchi's method.*—Light from a slit is brought to a focus by the lens or mirror to be tested. A short distance in front of (or behind) the focal plane is placed a line-screen having a spacing of 40–200 lines per centimeter, so oriented that the lines are parallel to the slit image. The shadow lines seen by the eye placed at the focus indicate clearly the characteristics of the wave surface as it leaves the lens or mirror. This method, which is described in Italian publications, is probably not so familiar to American readers as it deserves to be.

2. *Modification of Ronchi's method.*—Instead of a single slit, a line-screen illuminated by a frosted lamp or even by sky light may be used as the source, with the advantage that the illumination is so greatly increased that all observations can be made in a fully illuminated room. Also, for testing concave spherical mirrors, or lenses and paraboloids with parallel light, only one screen is required, one-half of which serves as the source, the other half as the observing screen. In this case the lines of the image are automatically parallel to those of the observing screen.

3. *A simple mounting* for the screen and light-source is described.

4. *Results of the examination* of a number of *lenses* and of a slightly defective concave *mirror* are given; these show that the method is fully as sensitive as the Foucault knife-edge test.

1. Of the many methods proposed for optical testing, two have come into very general use on account of their adequate sensitivity and simplicity in application. These are the Foucault knife-edge test and the Hartmann test. With either of these it is easy to observe an error of one-tenth of a wave-length in the wave surface as it leaves the principal plane of the instrument being tested, and under favorable conditions a considerably smaller error may be detected.

The knife-edge test is almost universally used by both amateur and professional opticians, because it requires only visual observations and practically no measurement. Consequently, it is qualitative rather than quantitative, although with a little practice the order of magnitude of the error can readily be estimated.

The Hartmann test, when used to the best advantage, requires one, or better two, photographs of the bundles of rays as they approach or depart from the focal plane and the subsequent measurement of the plates. It is, consequently, strictly quantitative and, for this reason, especially valuable to the user of an optical instrument who wishes to be fully aware of all its characteristics.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 386.

Several interference methods have been developed recently, among which may be mentioned those of F. Twyman[1] and A. A. Michelson.[2] In general, these methods require a rather complicated apparatus and a somewhat laborious series of observations. The results are, of course, highly accurate and strictly quantitative. About 1923 Vasco Ronchi announced a simple method of testing which is interferential in nature, but requires very little in the way of apparatus. The theory has been worked out by the discoverer,[3] and it is not the purpose of this communication to enter into that. We wish merely to call attention to a minor modification of his first published method and to some results which we have obtained in applying it to a few pieces of optical apparatus.

2. *Ronchi's method.*—In Figure 1 a slit S is illuminated by either white or monochromatic light from the source O, which, after passing through L, the lens to be tested, comes to a focus at F. A short distance in front of F is placed a coarse grating R, such as an ordinary half-tone screen, with its lines parallel to the slit S. If the eye is placed at or very near F, the lens will be seen as a bright circular disk crossed by a number of dark lines or bands, shown in Plates III and IV. The number of dark bands depends upon the distance from F to R, and is ordinarily equal to the number of lines on the screen R which fall within the light-cone from L. In some positions of R the bands are very faint, or appear to break up into a number of narrower bands; but in testing this is of no moment, for a position near F can always be found where they are well defined and of the proper number. If the lens is perfect, so that the light from all points on it comes to a focus precisely at F, the bands will appear perfectly straight and equidistant. But if a portion of the lens has a shorter focus, the light from this region comes to a focus between F and R, and the lines over it will be farther apart, while for a region of longer focus they will be closer together. If, however, R is placed outside the focus F, the phenomena are reversed, the area of shorter focus showing lines closer together than normal, and vice versa.

As an example, consider Plate III*f*, which shows the appearance

[1] *Philosophical Magazine* (6), **35**, 49, 1918.

[2] *Astrophysical Journal*, **47**, 283, 1918.

[3] *La prova dei sistema ottici*, Bologna, 1925. See also *Revue d'optique*, **5**, 1926.

of a concave spherical mirror when the screen R was placed 4.3 mm inside the center of curvature. Over a circular area in the center the bands are crowded together, thus indicating that the focus of this area is longer than that of the outer zone. The difference in the radii of curvature of the two portions was measured and found to be 0.016 inch. It is of interest to compute the resulting error for the central area. With the dimensions of the mirror given in a later section and $1\frac{1}{2}$ inches as the diameter of the central region, we find the maximum departure of the surface from the sphere defined by the outer zone to be one-sixth of a wave-length of green light.

3. *Modification of Ronchi's method.*—Ronchi found that the slit S need not be infinitely narrow, but may have a width such that its geometrical image is just equal to one of the opaque or transparent

FIG. 1

lines of the grating R. This suggested to us that for testing spherical mirrors at the center of curvature, or lenses and paraboloids with parallel light, the slit might be replaced by a portion of the screen R, backed by a lamp, as the source, while an adjacent portion would serve as the grating (see Fig. 2). Two distinct advantages are thus obtained: first, since light from many slits is used, the illumination is greatly increased; and, second, no adjustment for parallelism of slit and grating is required, since they are automatically parallel.

4. *Apparatus and observations.*—The optical set-up is shown schematically in Figure 2, and a sketch of the apparatus in Figure 3. Light from a small frosted lamp bulb is reflected by a mirror through the grating to the mirror or lens under test. It then returns through the same grating and enters the eye. As shown in Figure 3, the light-source, reflecting mirror, and grating are mounted in a tube which can be racked toward and away from the object under test; further, the grating can be rotated about the axis of the tube. Scales are provided for measuring focal lengths and the rotation of the grating. The lamp is an ordinary 2-volt flash-light bulb, taking current from a

lighting socket through a transformer. It is worthy of note that no attempt was made to darken the room in which the experiments were carried on. In fact, part of the time they were made on a bench by a window with direct sunlight falling upon the apparatus.

In considering the drawings shown on Plates III and IV it must be borne in mind that the disk of the Ronchi pattern is not perfectly sharp around the entire circumference. Because of overlapping of the various diffraction orders, the right- and left-hand edges of the disks are more or less blurred, the amount depending on the coarse-

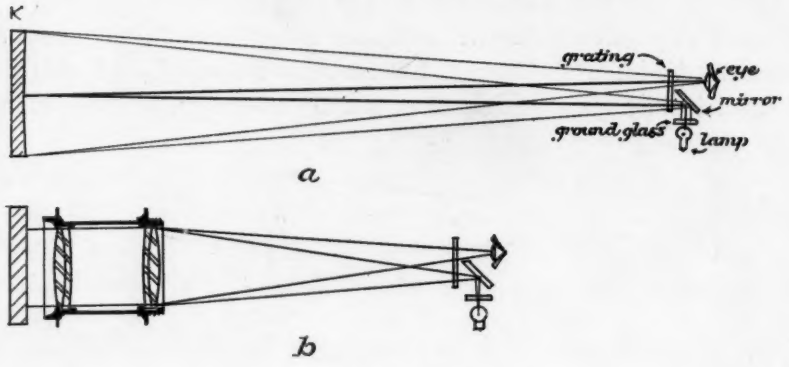
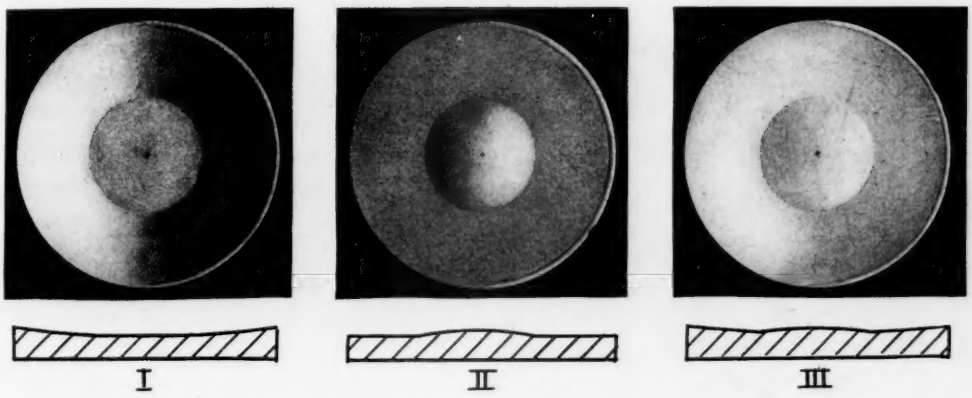


FIG. 2

ness of the grating used. The blurring does not, however, affect the interpretation of the Ronchi pattern and, because of the difficulty of reproducing it, has been omitted from the drawings. With this exception, the drawings are truthful representations of the phenomena observed.

Gratings of 100, 175, and 320 lines per inch were tried. That having the intermediate spacing was found the most satisfactory and was used throughout the tests. The mirror selected for examination is a spherical concave of $3\frac{3}{4}$-inch aperture and 36-inch radius of curvature. The surface is not perfect, but has a central area whose radius is 0.016 inch longer than that of the marginal region. Three drawings of the Foucault shadows for this mirror were first carefully made with the knife edge at and close to its mean center of curvature. These are shown in Plate II with their apparent sections drawn below them. To anyone familar with the Foucault method of

PLATE II

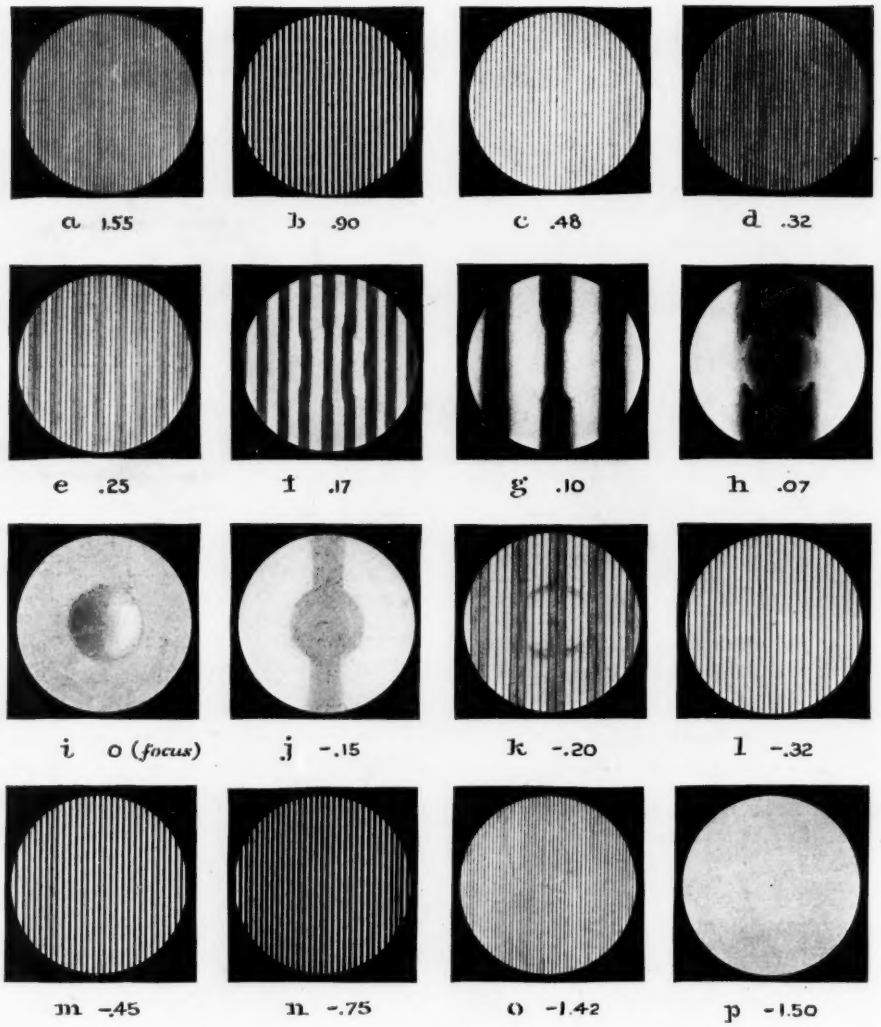


APPEARANCE OF A SLIGHTLY IMPERFECT SPHERICAL CONCAVE MIRROR AS SEEN UNDER THE FOUCAULT KNIFE-EDGE TEST

I. Pinhole and knife edge at center of curvature of zone *A* (central area).
II. Pinhole and knife edge at center of curvature of zone *B* (marginal zone).
III. Pinhole and knife edge at mean curvature of zones *A* and *B*.

The knife edge is advancing from the left. The difference of radii of curvature of the two zones is 0.016 inch, *A* being the longer.

PLATE III



APPEARANCE OF GRATING PATTERNS ON A CONCAVE SILVERED MIRROR, AS SEEN AT VARIOUS DISTANCES INSIDE AND OUTSIDE THE CENTER OF CURVATURE. THE DISTANCES ARE GIVEN IN INCHES

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testing an optical surface, these drawings will give adequate information as to the characteristics of the mirror.

Plate III shows the series of Ronchi band patterns or images seen when the grating (175 lines per inch) is placed at various positions near the center of curvature of the mirror, the distance inside or outside the center being given in inches beneath each drawing.

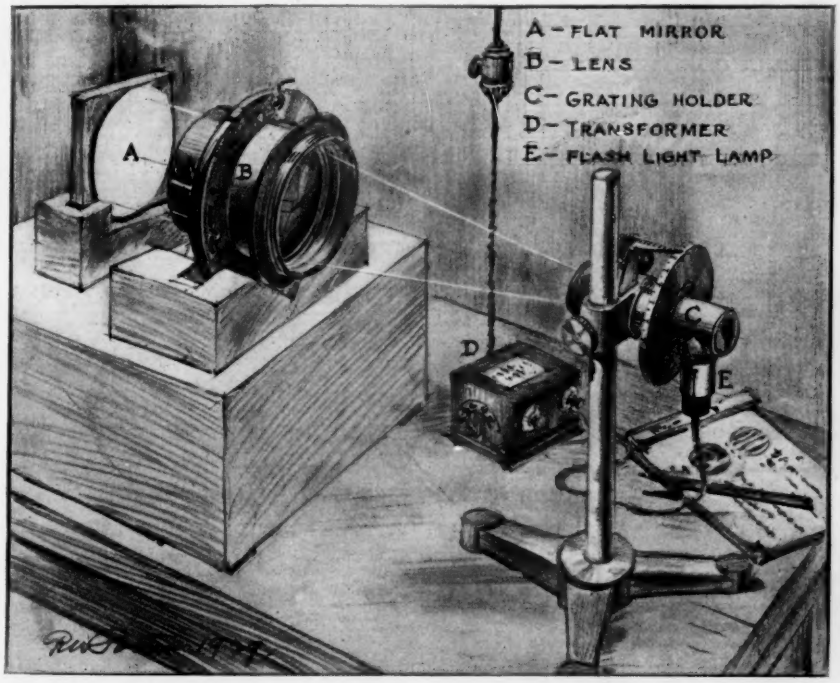


FIG. 3

Considered in order, *a* is the first noticeable pattern that appears as the grating is moved toward the center of curvature from the inside. The lines are faint and close together. At *b* another pattern appears with the lines more separated and intensely black. After another interval of confusion, *c* appears with the same separation as *b*, but with the faintness of *a*. At *d* the lines are crowded together again and are between *b* and *c* in intensity. The next pattern, *e*, shows a superposition of two line series, a coarse faint series of eight broad lines against about thirty-five sharp fine lines.

The grating now enters the region just inside the center of curvature which gives the most sensitive and satisfactory results for testing purposes. As the center is approached the bands spread out rapidly and reveal the features which indicate the character of the mirror surface. The positions illustrated are more or less arbitrary, as there are no intervals of confusion between *f* and *i* (center of curvature).

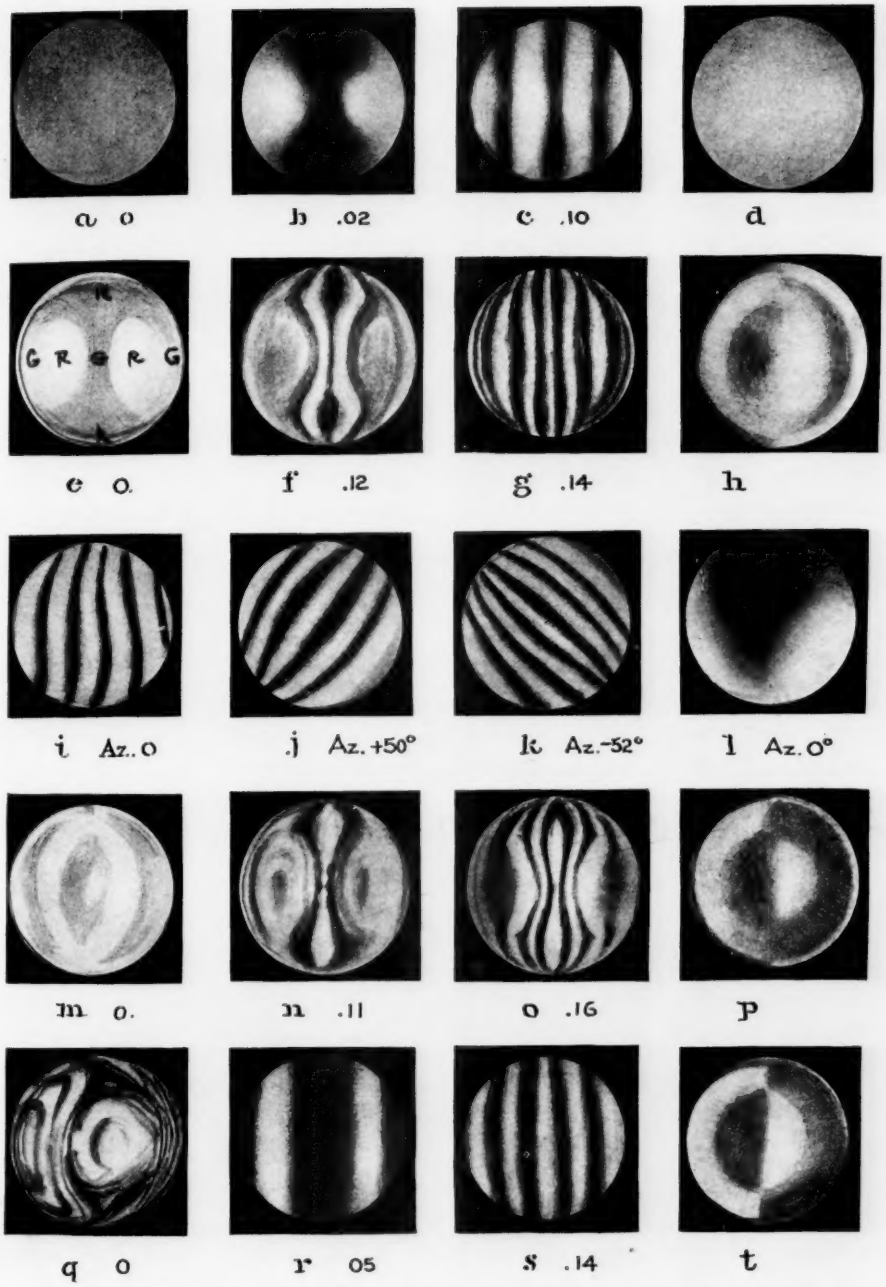
On passing outside the center of curvature the patterns appearing are those shown, and need no further description. After pattern *o* no more images appear, the mirror having a uniform surface, as at *p*. Returning to the region of greatest sensitivity, we note at *h* the peculiar appearance of the mirror's central area, the focal length of which is greater than that of the outer region. For this position of the grating (0.07 inch inside the center of curvature), the first wide band just covers the apparently raised central part of the mirror. Were this the only criterion for detecting surface errors, the interpretation would not be clear. But by moving the grating slightly nearer the center, the band continues to spread out until it covers the whole mirror; and if the grating is then shifted laterally so that the broadened band advances from the left, we see the shadow pattern *i*, which is exactly the appearance under the knife edge when the knife edge is at the center of curvature of the outer zone.

Hence for this particular position of the grating the shadows may be interpreted as in the "knife-edge test." The contour is determined by remembering that if the band advances from the *left* the resulting shadow models the surface of the mirror as though illuminated by light coming from the *right* at grazing incidence. On passing the center of curvature the first band appearing is very faint (*j*), and the additional bands rapidly fall off to a region of confusion.

Plate IV shows the results found by subjecting five different lenses to the Ronchi (or grating) test, with knife-edge shadows in the right-hand column for comparison. It should be remembered that these patterns are seen by the observer in color (secondary spectrum), while those obtained with a mirror are colorless.

The first test (*a*, *b*, *c*, *d*) is of a 4-inch telescope lens of ratio *f*/9. The lens at *a*, under the grating test, is uniform, like the knife-edge test at *d*, except that the left half is colored magenta and the right

PLATE IV



GRATING PATTERNS SHOWN BY FIVE DIFFERENT LENSES. THE LAST COLUMN GIVES KNIFE-EDGE SHADOWS FOR COMPARISON

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half green. These colors are reversed if the broad band advances from the right. By moving the grating 0.02 inch inside the principal focus, *b*, the first band narrows to the dumbbell form shown, and this pattern is repeated in a less pronounced way as more bands appear (*c*). The grating test thus reveals a lack of achromatism as well as a slight amount of spherical aberration not evident in the knife-edge test.

The second lens (*e*, *f*, *g*, *h*) was a camera objective, *f*/4.5, of 3-inch aperture. Here is strikingly shown at *f* and *h* the difference in appearance between the grating and knife-edge shadows.

Astigmatism is beautifully shown in the group of patterns appearing in the third row. This was a lens of the Petzval type of $2\frac{1}{2}$-inch aperture. With its lines vertical, the grating was placed so as to give *i*. Then, without changing its focal setting, the grating was rotated into the positions shown at *j* and *k*, giving four bands at *j* and six at *k*. The knife edge, when placed at the mean focus, confirms matters by showing the shadow *l*. The bands at *i*, *j*, and *k* strongly resemble the interference fringes produced by placing an astigmatic surface on one of pure revolution.

Another camera lens, *f*/4.5, aperture $3\frac{3}{4}$ inches (*m*, *n*, *o*, *p*), shows no trace of astigmatism and has characteristics similar to those shown in *e*–*h*. The appearance of this lens 10° off the axis is shown at *q*.

The results for the last photographic objective, *f*/5.6, aperture $2\frac{1}{2}$ inches, shown in *r*, *s*, and *t*, indicate clearly the fine optical quality of this lens.

The line-screens used in the foregoing observations were photographic copies of a commercial half-tone screen.

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A SPECTROSCOPIC STUDY OF THE VISUAL BINARY SYSTEMS 51 ξ SCORPII AND 2 η CORONAE BOREALIS

By Y. C. CHANG

ABSTRACT

The close binary system AB of *51 ξ Scorpii* has a period of 44 years. *The sign of the inclination* of its orbit is found to be negative. The *spectroscopic elements* as derived from Yerkes and Lick spectrograms are:

$$K=3.7 \text{ km/sec.}, \gamma=-29.4 \text{ km/sec.}, a \sin i=549{,}000{,}000 \text{ km},$$

$$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}=0.024 .$$

The fainter star of the visual binary system 2 η Coronae Borealis would appear to be the *more massive component*, as the ratio of the mass of the fainter component to the total mass is 0.53. *The inclination of its orbit* is negative. Its velocity-curve gives:

$$K=4.5 \text{ km/sec.}, \gamma=-6.0 \text{ km/sec.}, a \sin i=903{,}900{,}000 \text{ km},$$

$$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}=0.128 .$$

With sixteen visual binaries having determinate inclinations, *a statistical study* is made on *the orientation of their orbital planes*. The result of this investigation shows that there does not exist any noticeable parallelism among them and that they do not tend to meet in a common point. The data, though limited, point strongly to the conclusion of a *random distribution*.

From the spectrograms accumulated at the Yerkes Observatory and from published radial velocities of the Lick Observatory the writer was able to determine the signs of the inclinations of the visual binary systems 40 ζ Herculis and 6 β Delphini and to derive their spectroscopic elements at the same time.[1] In the present paper two more visual binaries, 51 ξ Scorpii and 2 η Coronae Borealis, are studied in very much the same fashion.

51 ξ SCORPII

51 ξ Scorpii ($=\Sigma$ 1998) is a triple system discovered by William Herschel in 1782. Its position for 1900 is $\alpha=15^h58^m.9$, $\delta=-11^\circ\ 6'$. According to the *Henry Draper Catalogue*, its spectral type is F8. The magnitudes for the primary are ptm. $=4.77$, ptg. $=5.27$, and those for the companion are ptm. $=5.07$, ptg. $=5.57$. The system has a proper motion of $0''.097$ in position angle $258^\circ.8$ (Auwers).

[1] *Astrophysical Journal*, **68**, 319, 1928.

The companion of the close binary system AB has completed about three revolutions since the time of its discovery. The circumstance that the components have about equal magnitudes has given rise to some confusion, and the orbits determined by different authorities show considerable variation from one another. Aitken's thorough investigation[1] proves that the companion had never been observed or measured except in the first or fourth quadrant.

TABLE I

YERKES SPECTROGRAMS OF 51 ξ SCORPII

Plate	G.M.T.	Obser. By	Meas. By	Quality	Vel.	Annual Mean	O−C
IIB 134.....	1907 May 28.692	B S	Ch	g	−33.9	−33.8	−0.8
B 725.....	June 17.717	F S B	Ch	p	31.8	.....	.....
IB 1020.....	1907 Apr. 13.809	Fox S	Ch	vg	35.3	34.0	−1.0
IB 1023.....	Apr. 19.832	F S	Ch	f	30.6	.....	.....
IB 1055.....	May 10.734	B S	Ch	g	37.1	.....	.....
IB 1062.....	May 12.570	Fox S	Ch	g	29.4	.....	.....
IB 1618.....	1908 May 8.754	L S	Ch	g	34.1	31.0	+0.9
IB 1625.....	May 11.815	L S	Ch	g	25.8	.....	.....
IB 6512.....	1922 May 22.751	σ S	Ch	p	29.0	30.0	−1.3
IB 6821.....	1923 Mar. 16.974	σ S	Ch	f	25.1	24.3	+4.3
IB 6827.....	Mar. 23.881	σ S	Ch	vg	21.5	.....	.....
IR 7736.....	1925 May 11.233	σ S	Ch	g	27.1	28.1	+0.5
IR 8072.....	1926 Apr. 8.428	σ	Ch	p	19.5	27.0	+1.6
IR 8106.....	May 4.234	σ	Ch	p	32.0	.....	.....
IR 8348.....	1927 Feb. 10.507	Hu S σ	Ch	vg	28.6	29.6	−1.2
R 1335.....	1928 May 12.280	σ S	Ch	f	30.6	−28.7	−0.3
R 1345.....	May 22.205	Hu σ S	Ch	f	−24.8	.....	.....

B = S. B. Barrett; S = F. R. Sullivan; F = E. B. Frost; Fox = Philip Fox; L = O. J. Lee; σ = Otto Struve; Hu = C. Hujer; Ch = Y. C. Chang.

The spectrographic observation of this star was begun at Yerkes in 1907. Up to the present seventeen well-measurable spectrograms, chiefly having the dispersion of one prism, have been accumulated. The Lick Observatory has published eight measures of spectrograms taken during the years 1899 to 1912.[2] The values of the radial velocities of Yerkes and Lick are listed in Tables I and II.[3]

In the column under "Annual Normal" of Table I a correction

[1] *Publications of the Lick Observatory*, **12**, 102, 1912.

[2] *Ibid.*, **16**, 234, 1928.

[3] Measures of radial velocity from four plates are found in the *Annals of the Cape Observatory*, **10**, Part 8, 212, 1928. Comparison with the adopted velocity-curve gives O−C: −0.7, −2.2, −4.7 for 1911, 1912, 1914. These velocities were not available when this work was carried out.

of -1.0 km/sec. has been applied to the Yerkes velocities to obtain better agreement with the Lick velocities.[1]

With the orbital elements derived from visual observations by Aitken[2] a velocity-curve has been computed. Its scale is adjusted to fit the spectrographic observations as well as possible. The comparatively large scattering does not warrant the computation of an orbit from the observed velocities. But they are quite adequate to determine the direction in which the radial velocity changes and thus settle the question regarding the sign of inclination of the orbit

TABLE II

ANNUAL NORMALS OF LICK RADIAL VELOCITIES

Date	Annual Normal	O−C
1899 June 20.....	−31.8	−3.2
1901 June 19.....	32.3	−3.0
1905 Apr. 12.....	34.8	+0.8
1907 May 11.....	35.2	−2.2
1910 July 14.....	32.5	−1.8
1912 Aug. 11.....	−33.1	−3.2

plane. However, preliminary spectroscopic elements are obtained from this curve and tabulated below side by side with the visual orbit:

Visual Elements	Spectroscopic Elements
$P=$ 44.70 years	$K=$ 3.7 km/sec.
$T=1905.39$	$\gamma=-29.4$ km/sec.
$a=$ 0".72	$a \sin i=549{,}000{,}000$ km
$n=$ 8°.054	$a=1{,}129{,}000{,}000$ km
$e=$ 0.75	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}=0.024$
$i=29°.1$	
$\omega=343°.6$	$\frac{m_2^3}{(m_1+m_2)^2}=0.217$
$\Omega=27°.2$	

We found from the curve (Fig. 1) that at the nodal point the primary is approaching with respect to the center of mass. Therefore, the inclination is positive ($M/p=10.49$).[3]

[1] *Publications of the Lick Observatory*, **16**, xxxi, 1928.

[2] *Ibid.*, **12**, 102, 1912.

[3] The formula involving the mass ratio and parallax is given in my former paper, *Astrophysical Journal*, **68**, 322, 1928.

Some determinations of the parallax of this star are given in Table III. The average of these five values with correction to the relative determinations gives the parallax of 51 ξ Scorpii as +0".046. Therefore, the mass ratio is

$$M=\frac{m_2}{m_1+m_2}=0.48, \text{ or } \frac{m_2}{m_1}=0.94 .$$

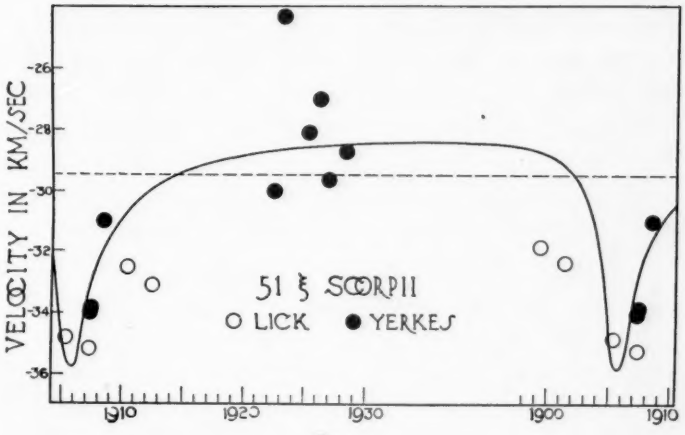

FIG. 1

2 η CORONAE BOREALIS

This star is a visual binary first observed by William Herschel in 1781. Its position for 1900 is $\alpha=15^h19^m.1$, $\delta=+30°39'$. The

TABLE III

Observer	Observatory	Parallax	Method
Lee, Joy, Van Biesbroeck...	Yerkes	+0".049±0.009	Photo.
S. A. Mitchell.....	Leander McCormick	.018± .010	Photo.
Lee, Joy, Van Biesbroeck...	Yerkes	.051± .006	Photo.
W. S. Adams.....	Mt. Wilson	.044	Spectr.
Keivin Burns.....	Allegheny	+0.041±0.005	Photo.

Henry Draper Catalogue gives the magnitudes for both the primary and the companion: ptm. = 5.58, ptg. = 6.14, for the primary, ptm. = 6.08, ptg. = 6.64, for the companion. Its stellar classification is Go.

The components of this binary system never come very close together. The measures by the Struve's cover more than a whole

revolution. Three revolutions have been completed since the binary system was first discovered. The best orbit is the one derived by O. Lohse,[1] using all observations up to 1916.

The fact that its spectrum shows variable velocity was first announced by O. J. Lee.[2] There are forty-six good spectrograms in the collection of the Yerkes Observatory, mostly of one-prism dispersion, covering a period of over twenty years. Nearly all of these have been measured by Dr. Otto Struve. Although the two components differ by only half a magnitude, the spectral lines show no noticeable broadening or doubling. Fitting the annual normals of the spectrographic observations to the velocity-curve (Fig. 2) computed from the visual elements, it is found that the scattering is small so that the direction of change of the radial velocity is unmistakable. Four annual normals are obtained from the published observations of the Lick Observatory. Both the Yerkes and the Lick observations are tabulated, with O−C for the annual normals in the last column, as shown in Table IV.

From the velocity-curve it is noticed that the primary is receding at the node with respect to the center of mass. The sign of the inclination is therefore negative. Upon measurement of the curve (Fig. 2) K is found to be 4.5 km/sec. The spectroscopic elements computed with this value of K are tabulated together with the visual elements derived by Lohse:

Visual Elements	Spectroscopic Elements
$P=$ 41.56 years	$K=$ 4.5 km/sec.
$T=1933.829$	$\gamma=-6.0$ km/sec.
$a=$ 0".89	$a \sin i=$ 903,900,000 km
$n=$ 8°.662	$a=1{,}060{,}000{,}000$ km
$e=$ 0.2721	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}=0.128$
$i=$ 58°.48	
$\omega=$ 217°.98	$\frac{m_2^3}{(m_1+m_2)^2}=0.207$
$\Omega=$ 25°.26	
$M/p=7.79$	

The determinations of parallax of this binary known to me are as shown in Table VI. These determinations, with correction applied, give an average absolute parallax of +0".066. The mass ratio

[1] *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, **20**, 119, 1909.

[2] *Astrophysical Journal*, **32**, 304, 1910.

TABLE IV

YERKES RADIAL VELOCITIES OF 2 η CORONAE BOREALIS

Plate No.	G.M.T.*	Obser. By	Meas. By	Quality	Vel.	Annual Mean	O−C
IIB 99.....	1907 Mar. 29.907	B S	L	g	−12.3	− 9.3	+0.3
			σ	.....	11.2	.....	.....
IB 1012.....	Mar. 31.923	P	σ	f	4.0	.....	.....
			σ	.....	7.8	.....	.....
IB 1021.....	Apr. 13.871	Fox S	L	g	8.0	.....	.....
			σ	.....	9.8	.....	.....
IIB 123.....	May 27.640	F S	L	g	10.9	.....	.....
			σ	.....	10.5	.....	.....
IB 1544.....	1908 Mar. 20.926	B S	L	g	3.0	− 5.3	+4.1
			σ	.....	4.0	.....	.....
IB 1563.....	Apr. 11.890	L S	L	g	4.3	.....	.....
			σ	.....	7.3	.....	.....
IB 1611.....	May 2.814	B S	L	g	6.2	.....	.....
			σ	.....	6.8	.....	.....
IB 2383.....	1910 June 20.630	L S	L	f	12.3	−15.6	−6.3
			σ	.....	19.7	.....	.....
			Ch	.....	17.7	.....	.....
IB 2427.....	July 18.617	L S	L	g	20.4	.....	.....
			σ	.....	11.7	.....	.....
			Ch	.....	12.0	.....	.....
IB 4397.....	1916 Jan. 31.942	S H	σ	g	2.0	− 3.7	+3.8
			Ch	.....	5.3	.....	.....
IB 5700.....	1920 Mar. 5.838	Bk S	σ	f	1.9	− 7.0	−1.0
IB 5720.....	Mar. 15.880	Bk S	σ	f	7.5	.....	.....
IB 5752.....	Mar. 30.810	Bk S	σ	g	3.7	.....	.....
IB 5815.....	May 21.646	B S	σ	f	7.3	.....	.....
IB 5829.....	May 31.658	Bk S	σ	p	14.6	.....	.....
IB 6468.....	1922 Apr. 7.761	σ S	σ	vg	8.7	− 6.0	−1.0
IB 6511.....	May 22.708	σ S	σ	f	5.5	.....	.....
IB 6533.....	June 2.637	σ S	σ	g	3.9	.....	.....
IB 6860.....	1923 Apr. 27.873	σ S	σ	g	14.0	− 1.7	+2.6
IR 6922.....	June 8.755	σ S	σ	g	0.5	.....	.....
IR 6929.....	June 11.752	σ S	σ	g	0.8	.....	.....
IR 6947.....	June 18.700	σ S	σ	f	4.3	.....	.....
IR 6956.....	June 22.719	σ S	σ	g	1.4	.....	.....
IR 6976.....	July 6.670	σ S	σ	f	0.1	.....	.....
IR 6980.....	July 9.762	σ S	σ	p	1.7	.....	.....
IR 7018.....	July 30.608	σ S	σ	g	1.1	.....	.....
IR 7025.....	Aug. 6.599	σ S	σ	g	0.5	.....	.....
IR 7029.....	Aug. 8.579	B S	σ	p	− 5.4	.....	.....
IR 7046.....	Aug. 18.641	σ S	σ	g	+ 7.7	.....	.....
			Ch	.....	− 1.3	.....	.....
IR 7069.....	Aug. 25.641	σ S	σ	f	+ 3.9	.....	.....
IR 7256.....	Dec. 17.014	σ S	σ	g	− 1.1	.....	.....
IR 7272.....	1924 Jan. 7.014	σ S	σ	g	2.9	− 3.5	+0.5
IR 7279.....	Jan. 13.014	B S	σ	g	4.3	.....	.....
IR 7289.....	Jan. 20.016	B S	σ	p	− 8.4	.....	.....
IR 7295.....	Jan. 20.997	σ S	σ	f	+ 1.6	.....	.....
IR 7744.....	1925 May 18.207	B S	σ	g	− 3.8	− 3.8	−0.6
IR 8093.....	1926 Apr. 20.372	σ S	σ	f	− 4.7	− 3.8	−1.1
IR 8094.....	Apr. 26.190	σ S	σ	g	+ 1.2	.....	.....
IR 8097.....	Apr. 30.197	σ	σ	f	− 2.4	.....	.....

TABLE IV—*Continued*

Plate No.	G.M.T.*	Obser. By	Meas. By	Quality	Vel.	Annual Mean	O−C
IR 8104.....	1926 May 4.108	σ	σ	g	7.2	.....	.....
IR 8199.....	May 11.234	B S	σ	g	5.7	.....	.....
IR 8347.....	1927 Feb. 10.492	Hu S	Ch	g	3.1	− 2.0	+0.1
IR 8377.....	Feb. 24.493	Mgn S	Ch	vg	0.5	.....	.....
IR 8385.....	Mar. 4.328	σS Hu	Ch	vg	− 2.3	.....	.....
R 1334.....	1928 May 12.174	σ AP	Ch	p	+ 0.6	− 1.2	+0.2
R 1338.....	May 13.149	σ AP S	Ch	g	− 2.9	.....	.....

* G.M.T. before 1925; U.T. after 1925.

Bk = Dorothy W. Block, AP = A. Pogo, Mgn = W. W. Morgan.

is $m_2/(m_1+m_2)=0.525$. If this is real, the fainter component would be more massive than the brighter component.

Among the four systems discussed in this paper and the previous one, the two components of ζHerculis as well as those of β Delphini

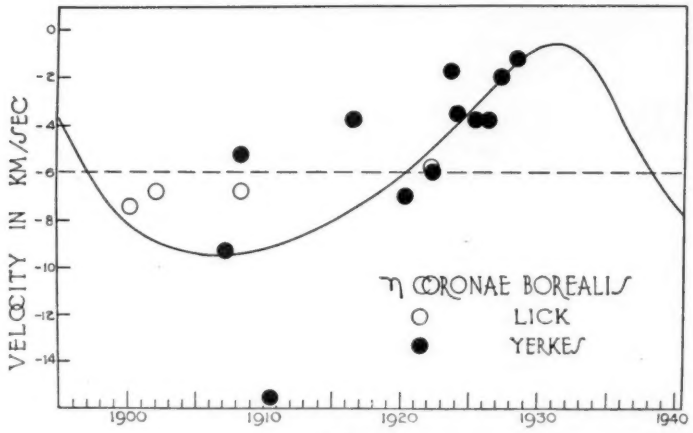

FIG. 2

differ in brightness by over a magnitude. We naturally expect that only the brighter component would leave its imprint on the spectrogram. But the difference of magnitude amounts only to 0.3 in the case of ξ Scorpii and 0.5 in the case of η Coronae Borealis. Although no doubling or widening of the spectral lines has become noticeable, it is possible that the spectrum of the fainter companion may affect the accuracy of the determination of radial velocity to a certain extent. It is therefore not surprising to find some comparatively

large scattering in the individual measurement of radial velocity of these two systems. However, the consequence of such masking, if any, could only reduce the range of the velocity-curve; its character or the position of the maximum and minimum on the curve would

TABLE V

ANNUAL NORMALS OF LICK RADIAL VELOCITIES

Date	Annual Normal	O−C
1900.35	−7.4	+0.8
1902.13	6.8	+2.1
1908.40	6.8	+2.6
1922.17	−5.8	−0.7

not be altered. That is to say, in these two systems the spectroscopic elements derived may need some systematic correction, while the determination of the sign of inclination is conclusive.

In any statistical study of the orientation of the orbital planes of the visual binaries Aitken states that "the fact that the orbit-

TABLE VI

Observer	Observatory	Parallax	Method
F. Slocum	Yerkes	+0".073±0.014	Photo.
W. S. Adams	Mt. Wilson	.069	Spectr.
J. O. Miller	Sproul	.085± .008	Photo.
W. S. Adams	Mt. Wilson	.055	Spectr.
S. A. Mitchell	Leander McCormick	+0.047±0.012	Photo.

elements do not define the plane uniquely presents a serious difficulty."[1] It is therefore very desirable to make such a study with binary orbits having determinate inclinations. Among well-determined orbits of double stars ten systems are found of which the sign of the inclinations is known. They are: 13 Ceti, Capella, Sirius, Procyon, 9 Argus, 11 ϵ Hydrae, 53 ξ Ursae Majoris, α Centauri, 70 Ophiuchi, 85 Pegasi. There are two well-known systems, 16 ζ Cancri and 42 Comae Berenices, having their orbital planes perpendicular and parallel to the line of sight, respectively. The signs of their inclinations are therefore immaterial. Adding to these the

[1] Aitken, *The Binary Stars*, p. 218.

four systems discussed by the writer, we have a total of sixteen visual binaries with determinate inclinations as our data for the statistical investigation. If the amount of material is rather scanty, it can be said that the orbits employed are free from the ambiguity inherent in the previous investigations along this line.

The present study makes special reference to two questions: (1) Whether these orbital planes show any parallelism to the Milky Way or some other plane. (2) Whether they tend to intersect at a certain point, for instance, the center of our galaxy.

By plotting the poles of these sixteen orbits in both the equatorial and galactic co-ordinates it is found that they do not show any noticeable concentration.

To settle the second question, equations of the orbital planes of these sixteen binaries were formed and the common point of intersection was obtained by solving the sixteen equations with least squares. The perpendicular distances from this supposed point of intersection to the individual planes are found to be of the same order of magnitude as those among the binaries themselves. From this fact we may safely conclude that they do not indicate any tendency to meet in a common point.

The result obtained here points strongly to a random distribution in the orientation of the orbital planes, as several previous investigators have concluded from a similar study with orbits having ambiguous inclinations. A paper to be published in the *Astronomical Journal* will present this investigation in greater detail.

YERKES OBSERVATORY
August 1929

MINOR CONTRIBUTIONS AND NOTES

THE ABSORPTION SPECTRUM OF CS_2 IN THE NEAR ULTRA-VIOLET

In a recent article with this title,[1] E. D. Wilson reported measurements and estimated intensities of a large number of absorption "lines" of carbon disulphide vapor. Some time ago Professor H. A. Barton and I photographed this spectrum in the first and second

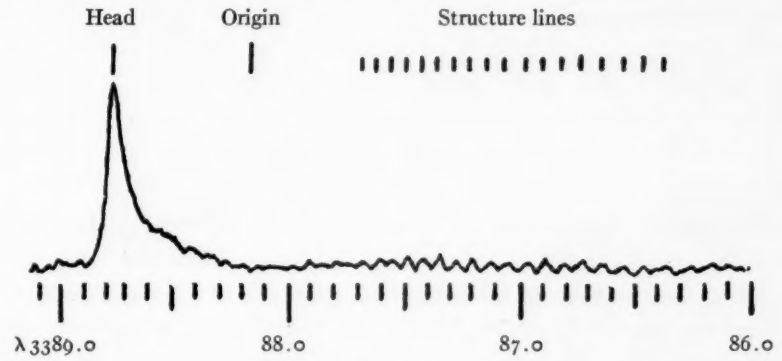

FIG. 1

orders of the Harvard 21-foot grating, which gives much greater dispersion and resolution than the Hilger E-1 spectrograph used by Wilson. Our work was discontinued, however, because of the apparent impossibility of finding any regularities of theoretical importance in this rich and complicated spectrum. Of band systems belonging to polyatomic molecules, that of formaldehyde studied by Henri and Schou[2] appears to be the most promising in this respect.

The grating plates show the more intense part of the CS_2 spectrum as an intricate complex of lines and bands in which it is difficult to find a given structure repeated. At the less refrangible end,

[1] *Astrophysical Journal*, **69**, 34, 1929.

[2] *Zeitschrift für Physik*, **49**, 774, 1928.

however, there are a number of weaker bands with sharp heads, shaded to the side of high frequency. The majority of these show a rapid fall of intensity from the head, followed by a slight rise, merging into a series of very finely spaced lines. Figure 1 shows a photometer curve of the band with its head at λ 3388.78; the curve was taken with the new Zeiss recording photomicrometer in this Laboratory. The distribution of intensity as a whole closely resembles the *P*- and *R*- branches of a band in the spectrum of a diatomic molecule when the source is at a low temperature. Assuming this simple structure, and taking the intensity minimum as the position of the origin, we find by a rough calculation from measurements of the foregoing band for the moment of inertia of the molecule in its upper and lower states: $I' = 205 \times 10^{-40}$ and $I'' = 247 \times 10^{-40}$ gram cm^2. If the molecule were of the linear form *S-C-S*, this would give for the separation of the *S*- and *C*-nuclei in the lower state 1.53×10^{-8} cm. One can obviously not attach much significance to these figures until a more complete theory of polyatomic molecular spectra is developed.

In looking for regularities in the spacing of the bands, it would seem desirable to take into account the appearance of the individual members, since presumably bands of the same series should have a similar structure. This is not the case for any of the series proposed by Wilson. For example, the member $\nu = 29{,}511.6$ of the first series in his Table II is apparently a measurement of the center of gravity of the group of lines shown in Figure 1 (above), which was unresolved on his plates. The adjacent members do not resemble this at all. This series, "more satisfactory" than the other two given in Table II, is further vitiated by two numerical errors (in the seventh and twenty-first differences, $\Delta\nu$) each of which throws a portion of the series off by 100 cm^{-1} with respect to the rest. Furthermore, there are frequent gaps in the other two series, and none shows a trend in the $\Delta\nu$'s similar to those in the spectra of diatomic molecules, although such a trend is claimed by the author. Hence the reality of these series seems open to serious question. On the other hand, a study of all the bands in this region which show the structure described above reveals a surprising number of frequency differences lying between 250 and 280 cm^{-1}, and a strong sequence of

six heads, $\nu = 29{,}336.7$, 29,081.3, 28,815.9, 28,543.5, 28,270.6, and 27.986.9 cm^{-1}, is particularly prominent. In the more intense part of the system the short series and the few constant frequency differences noted by Wilson may well be fortuitous, in view of the extreme richness of the spectrum in this region. For solving the vibrational structure of this band system, spectra taken with extremely low vapor densities should prove helpful, since these show only a few of the strongest bands near the middle of the group. It seems probable that further progress along this line will have to await a more detailed study of the infra-red absorption of CS_2.

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September 1929

REVIEWS

Statistical Mechanics. By R. H. FOWLER. Cambridge University Press, 1929. Pp. viii+507. Figs. 28. 35*s*.

The author of this monumental work, R. H. Fowler, was the writer of the Adams prize essay for 1923–1924. This essay (on the physical state of matter at high temperatures) forms the basis for his *Statistical Mechanics*. The book is an extensive treatment of the equilibrium theory of statistical mechanics; or, as the subtitle excellently describes it, *The Theory of the Properties of Matter in Equilibrium*. The student of celestial mechanics in his study of periodic orbits can well appreciate the complexities that arise in an investigation of dynamical systems which are collections of large numbers of atoms, molecules, or electrons. It is not only astonishing that a statistical theory is possible for such systems, but it is also remarkable that the resulting theory is of such enormous practical importance in the domains of physics, astrophysics, and chemistry. Despite the great need for them, there has been a lack of texts approaching anywhere near to a complete treatment of present knowledge of the equilibrium properties of assemblies of atoms, molecules, or electrons. However, Professor Fowler's enormous volume of 570 pages and 1607 numbered equations fully eliminates this deficiency. It most assuredly deserves to be ranked with the standard classical treatments of statistical mechanics by Gibbs, Boltzmann, Jeans, and Ehrenfest, as the particular representative of the quantum statistical mechanics.

It has been the practice in the majority of texts on the quantum theory and statistical mechanics to develop first the classical theory and then to make whatever modifications may be demanded by the quantum mechanics. Contrary to this procedure, Professor Fowler incorporates the quantum conditions at the beginning of his monograph and in each problem obtains first the formulae of quantum statistical mechanics. The laws of classical statistical mechanics are then derived as limiting cases of the quantum mechanics. Thus Professor Fowler obtains a development that actually succeeds in giving to quantum statistical mechanics the appearance of a general theory which includes the older statistical mechanics as a special case. Furthermore, his book proves that a presentation of a subject not in the same order as the historical development may still be logical.

The book starts with a comparatively short Introduction in which the author states the fundamental assumptions of statistical mechanics. "Normal" properties he defines as those which the assembly possesses on a time average, and he wisely makes his discussion of the passage from "time averages" to "space averages" very brief. Curiously, the term "weight" is used in place of a priori probability; this to avoid implying that the behavior of the systems is according to the laws of chance and not to dynamical or other determinate laws in ordinary space and time. Expected or average values are calculated rather than, as is often done in statistical mechanics, most probable values. Professor Fowler points out the ease and rigor of mathematical presentation of the former over the latter—a fact that has long been appreciated in other applications of the theory of probability. The first chapter closes with a brief development of Liouville's theorem.

The second chapter contains the rules which are to be employed in determining the weights for both classical and quantized systems. The general conservative dynamical system or surface ensemble, generalized to include quantized systems, is here used instead of the canonical ensemble of Gibbs; though, according to Professor Fowler, the latter has advantages of logical precision, the former seems physically the more natural. The older form of the quantum theory of atomic systems is employed, a procedure satisfactory for 1926, as Professor Fowler points out in the Preface of the book, but hardly one to be recommended at the time of the appearance of the book in 1929. The reviewer feels, however, that it is just as important to have such a unified treatment of statistical mechanics on the older quantum theory as to have a development of the classical statistical mechanics. The wonderful success of the new mechanics should not make us forget that the older quantum theory was not a postulate without merit, as is evident from reading Professor Fowler's book. In fact, Professor Fowler points out that only in exceptional cases are the primary changes of the new mechanics of practical significance in statistical mechanics, this despite their theoretical importance. Furthermore, in the main body of the monograph he indicates how the new mechanics may be regarded as merely a change of weights and energies of the stationary states, and he devotes the last chapter of the book to the new statistical mechanics.

The second chapter contains also a derivation of the Maxwell-Boltzmann law by a very ingenious method originally due to C. G. Darwin and Fowler. The mathematical scheme is fundamental throughout the entire monograph. Briefly, the procedure is to identify those sums of factorial

products arising in the computations of average values with certain coefficients in suitable multinomial expansions. In this respect the method is similar to that employed in the computation of expected values by the so-called method of arrays (Darwin and Fowler use the term "partition function" in place of "array") in the theory of probability. However, they proceed next to obtain an approximate value for these coefficients by expressing them by means of Cauchy's integral of complex variables and evaluating the integral by the method of steepest descents. Although there are other satisfactory methods of deriving approximations for these coefficients, such as solving a simple integral equation, upon once evaluating a general integral, Professor Fowler arrives at a simple formula that enables him quickly to deduce approximate values for those sums that arise in his work.

With the aid of this mathematical tool Professor Fowler proceeds in the first part of the book to develop the ordinary equilibrium theory for perfect gases, crystals, and any general body obeying classical laws, including a treatment of radiation, applications to specific heat of gases, and the properties of simple crystals. One chapter is devoted to all types of dissociation and evaporation, another to a very critical discussion of the relationship of the equilibrium theory to classical thermodynamics. This is followed by a chapter on reactions at low temperatures and Nernst's heat theorem, the latter being derived as a theorem in pure statistical mechanics. Three chapters are given to a very complete discussion of imperfect gases; one of them, headed "Interatomic Forces," was contributed by T. E. Lennard Jones. Professor Fowler attempts to cover the entire equilibrium theory of thermionics. Here the most important part of his work is the derivation of a theoretical formula giving the vapor density of free electrons in equilibrium with a hot metal, including the effect of the space-charge. Applications are made in subsequent chapters to magnetic and dielectric phenomena of matter, and properties of dilute solutions along with the theory of Debye and Hückel.

Three chapters of the monograph are devoted to applications of the equilibrium theory, to stellar phenomena under the conditions of high temperatures, that is, the applications proposed by the examiners for the Adams prize. First the equilibrium theory of a gas of highly ionized atoms is developed, making certain assumptions as to approximate form so as to include the effects of the sizes of the ions and their electrostatic fields. Then a start is made in the investigation of the properties of stellar material in the interior of a star, although here the author is rather brief and careful to avoid duplication of the work of Eddington. On the other hand,

the discussion of atmospheric problems is almost exhaustive and it includes the equilibrium theory of an isothermal ionized atmosphere and the permanent electrical fields and charges in it, the statistical theory of absorption lines, the escape of molecules from an atmosphere, and formation of the chromosphere and expulsions of atoms by radiation pressure. From the relative intensities of the absorption lines of any star may be obtained some indication of the relative number in the various stages of ionization in the reversing layer, and therefore of the temperature in this region. This important idea was first successfully applied by Saha and the method refined by Fowler and Milne, who suggested the use of maximum intensities. Numerical calculations of maxima, where observational material was available, are given to adapt the theory to astronomical observations.

The author does not confine himself entirely to equilibrium theory, and he includes in his monograph chapters on mechanisms of interaction, radiative processes, fluctuations, and a final chapter on the new mechanics. Though the book is not intended for the beginner in statistical mechanics, one may read the last chapter on the new statistical mechanics without a knowledge of the new quantum mechanics. This last chapter is really a splendid presentation of the Fermi-Dirac and Einstein-Bose statistics. The astronomer would here be interested in Professor Fowlers' attempt to solve the difficulty in the theory of the ultimate fate of the white dwarf stars by means of the Fermi-Dirac statistics.

WALTER BARTKY

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Grundriss der Astrophysik. BY KASIMIR GRAFF. Leipzig-Berlin: B. G. Teubner, 1928. 8vo. Pp. viii+751. 468 illustrations and 6 large plates. R.M. 42.60 unbound; R.M. 45.–bound.

The progress of astrophysics in the last decade has been so rapid that a number of excellent books published comparatively recently have grown so hopelessly out of date that the publishers have found it advisable to issue completely new texts rather than to bring the old editions up to date by suitable inserts and footnotes. We learn from the Preface that the volume under review is intended to replace J. Scheiner's well-known *Populäre Astrophysik* originally published in 1908 and issued in a second edition by Scheiner in 1912. After the death of Scheiner in 1913, Dr. Graff began the preparation of a third edition which appeared after the war, in 1922. In order to reflect recent progress in astrophysics

Dr. Graff has now found it expedient to re-write the whole book and to assume full responsibility for it. In volume as well as in quality the new book not only reverts to pre-war standards but it even surpasses the pre-war editions of Scheiner. The total number of pages, which was 718 in the first edition and which had been reduced to 459 in the third edition (of 1922), has now been increased to 751.

In the Preface the author states that he has attempted to cover only the well-established facts in astrophysics, leaving out the more or less hypothetical speculations which abound in modern work. This was perhaps a wise decision in so far as it will prevent the book from growing out of date too rapidly. On the other hand, it can hardly be denied that a limitation of this nature deprives the work of that fascination which we find in all new developments in science. The reader is not actually introduced into the astronomer's workshop, and he does not share with him the delights and disappointments that accompany the real research. The reviewer would have preferred to see more of the unfinished problems included, even at the risk that part of this work may turn out to be in error. However, this is a matter of taste, and as it stands the new book can be very heartily recommended as an excellent summary of the more conservative type. Being intended as an "Outline" (*Grundriss*) it does not presuppose any particular knowledge in physics and mathematics, and it should be within the grasp of the beginner in astronomy. More advanced students will find it a convenient reference book.

The personality of the author has impressed itself upon the whole character of the work. Dr. Graff, who is now director of the observatory at Vienna, is well known for his researches in the fields of photometry and of the surface-structure of the planets. Doubtless the most interesting and important part of his book deals with the methods and results of stellar photometry. This portion will be of great interest to all those who are actively engaged in photometric work. Considerable attention has also been devoted to the solar system. The spectroscopic results might well have received a more detailed treatment, but this would have necessitated considerable expansion in the introductory part on the physical foundations. Theoretical discussions have in general received but little attention.

A large work of this general character is bound to have imperfections. A few minor points may be mentioned at this time. On page 503, discussing the spectral classification of the stars, the author explains the use of the letter k; this is obviously based upon a misunderstanding. On page

556 we find the statement that the B and A stars of great absolute magnitude are systematically whiter than the absolutely faint stars. This is somewhat misleading. As Hertzsprung has shown, there is a pronounced tendency for the absolutely bright stars to have a lower color-temperature than the faint stars and the effect is in the same direction as for the later spectral types, although its explanation may be quite different. The term "whiter," as used by the author, is correct only in the sense that the bright stars are less bluish than the faint ones of the same spectral class. In discussing the *Gegenschein* on page 484 the mechanical hypothesis of Gyldén and of F. R. Moulton might well have been mentioned.

The number of misprints is very small, but a few occasional slips in the spelling of foreign names have been noticed. The printing is excellent and the paper is of very good quality. The illustrations are numerous and well chosen. The full-page plates add a great deal to the attractiveness of the book. The close-up view of the 100-inch reflector at Mount Wilson is particularly striking and gives a good idea of this mechanical wonder. The frontispiece contains a general view of Mount Wilson with the dome of the 60-inch reflector and the two tower-telescopes in the foreground.

OTTO STRUVE

A Star Atlas and Reference Book (Epoch 1920) for Students and Amateurs. By ARTHUR P. NORTON. 4th ed., enlarged London: Gall and Inglis, 1927. 22×29 cm. Pp. viii+54. Pls. 19. 10*s.* 6*d.*

This atlas is recommended unreservedly. Its eighteen star maps are excellently clear, neither unsightly because of overexaggerated star-images nor confusing by reason of the inclusion of many faint stars. The lists of interesting telescopic objects accompanying the maps are adequate. There is, in addition, a sketch map of the moon with the names of the important formations engraved thereon. Some forty-seven pages of text are devoted to nomenclature, the solar system, spectroscopy, observing, care and use of the telescope, and many useful tables.

With two exceptions it is all that such an atlas should be—it has no guide to pronunciation and no table of stellar distances. If the names of the constellations and of the bright stars are daily mangled in the halls of the professional, what can be expected of "the student and the amateur" for whom, we are told, the book was compiled! It is not too much to say that an array of Arabic, Greek, and Roman astronomical names

without a glossary of pronunciation has caused to falter many a would-be delver into the science. As to stellar distances, a short table would suffice if it included only the very bright stars, with parallax known or unknown, and some of the very near stars.

The use of the English "billion" where almost all nations would use "trillion" is regrettable, because of the uncertainty it causes. It would be better to say "a million million." A few unimportant corrections are sent directly to the publishers.

STORRS B. BARRETT